INVESTMENT, THE GOODS MARKET AND THE IS CURVE

UNIT 4

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4.1. THE INVESTMENT FUNCTION

Until now we have considered that investment (I) is a constant value, which is given in the economy, that is, we have considered that I is equal to autonomous investment ($\bar{I}$). However, we are now going to consider that I depends on the interest rate. Thus, we define the investment function as the relationship between investment and the interest rate. In order to invest, we can assume that companies borrow money that must be repaid with interest. Therefore, as interest rates are higher, companies will want to borrow less money, as the cost of borrowing money will be higher. Alternatively, we can assume that the company can finance itself and use its own money to make its investments. In this case, the interest rate can be considered an opportunity cost, since, instead of investing, the company could take the money and lend it to third parties, thus charging interest. The higher the interest, the more the company will earn from lending its own money, and the less willing it is to use it to finance its investments. Thus, if the interest rate rises, investments will fall.

Even so, companies may invest more, or less, depending on other circumstances. For example, from the expectations of their business. If companies think that the business they are going to do is very good, they will be willing to invest more, regardless of how high or low the interest is.

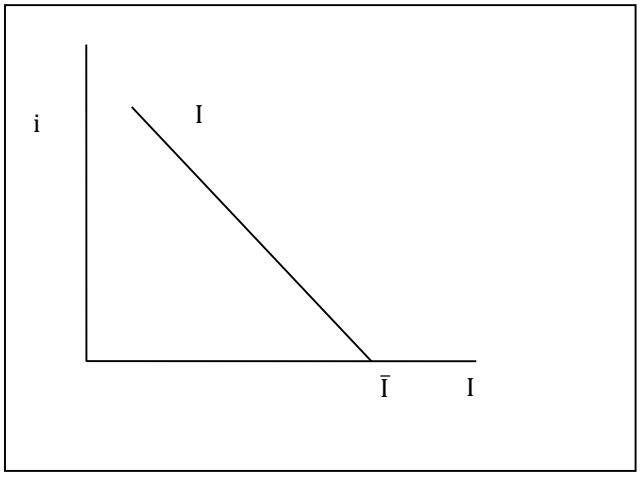
In this way, we can easily define the investment function as follows

$$I = \bar{I} - bi \qquad b>0$$

where $\bar{I}$ is autonomous investment, i is the interest rate, and b is a parameter which shows the sensitivity of the investment to the interest rate, that is, b shows how much investment decreases if the interest rate increases by one unit.

We can graphically represent this function in the (i, I) plane. Figure 1 shows the investment function.

Figure 1. Investment Function



If the interest rate is zero, the $I = \bar{I}$. However, as the interest rate begins to rise, the value of I will begin to decrease. The slope of the curve is then negative. What does it depend on? Usually, the dependent variable of a mathematical function is plotted on the vertical axis, and the independent variable on the horizontal axis. This is why the function $I = \bar{I} - bi$ does not correspond exactly to the curve represented in Figure 1. The function represented by the curve in Figure 1 can easily be obtained by clearing the interest rate from the previous expression. Thus, as

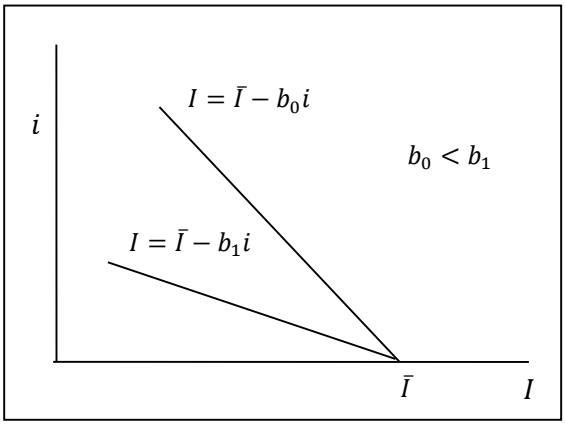
$I = \bar{I} - bi$, if we clear i, we obtain $I - \bar{I} = -bi$ and therefore

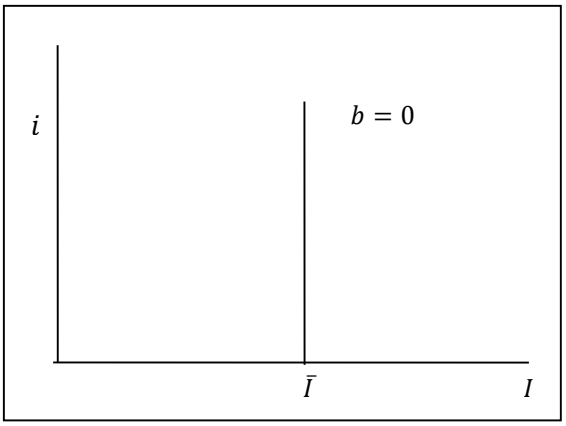
$$-\frac{1}{b}I + \frac{1}{b}\bar{I} = i$$

The slope of the investment curve is - $\frac{1}{b}$. Thus, as the value of b increases, the less the slope will be, that is, the curve will be flatter. As long as its value is smaller, the slope will be greater. In the left graph of Figure 2, we represent the effect of an increase in the value of b on the curve of I. As b is larger, the curve becomes flatter. In the right graph of Figure 2, we represent the specific case that b is zero. We will have a totally vertical investment curve, then $I = \bar{I}$. Can there be a situation in which the investment does not depend on the interest rate? We can imagine the following situation. If banks do not have money to lend, then, even if entrepreneurs go to the bank for borrowed money, the bank will not be able to lend them money if they do not have it. Thus, when there are liquidity constraints, the money demand curve will be totally vertical. In this case, the investment will be independent of the interest rate, and it will be vertical. On other occasions, the bank may be reluctant to lend because the risk they incur in lending the money is higher than the market rate. That is, the bank will not lend money when

it thinks that the entrepreneur's business is going to fail, or when the risk of its failure is greater than the return that could be obtained by lending the money. In these two cases, the investment will not depend on the interest rate, and the curve will be vertical.

Figure 2. Effect of *b* on the investment curve





On the other hand, it is also important to note that autonomous investment affects the position of the investment curve. Increases of $\bar{I}$ shift the investment curve to the right. Expectations can influence autonomous investing. When expectations are good, entrepreneurs will want to invest more, regardless of the value of the existing interest rate.

4.2. INVESTMENT, AGGREGATE DEMAND AND EQUILIBRIUM IN THE GOODS MARKET

As we have seen in the previous unit, the AD is determined by the sum of consumption, public spending on goods and services, and investment. We can now include our investment function in the AD curve that we have already obtained in that unit.

Given the fiscal policy, that is, once we have defined the levels of spending, transfers and the tax rate, we know that consumption is equal to

$$C = \overline{C} + c(Y + \overline{TR} - tY) = \overline{C} + c\overline{TR} + c(1-t)Y$$

Investment has been defined by the investment function. In this way, now

$I = \bar{I} - bi$

The level of public spending is defined by fiscal policy. Thus

G = $\overline{G}$

In this way

$$AD = \overline{C} + c\overline{TR} + c\,(1\text{-}t)\,Y + \bar{I} - bi + \overline{G}$$

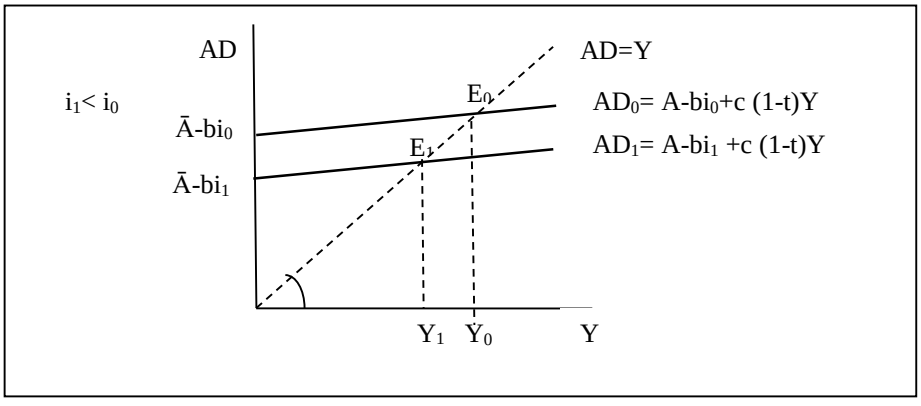
Thus

$$AD = \overline{A} - bi + c\,(1\text{-}t)Y\,,$$

where autonomous spending $\overline{A}$ is equal to $\overline{A} = \overline{C} + c\overline{TR} + \bar{I} + \overline{G}$

The AD now depends on the value of the interest rate. If the interest rate increases, the AD will decrease. Whereas, if it decreases, its value will increase. The effect of the interest rate on the AD can be represented with the income-expense model. Figure 3 shows how the interest rate affects the AD and the equilibrium in the goods market. The value of the interest rate affects the value of the AD at origin. If the interest rate is i_0, for an income level equal to zero, the AD will be equal to $AD = \overline{A} - bi_0$. As income increases, given the positive slope of the AD curve, its value will begin to grow. As we saw in Unit 2, the equilibrium point is obtained where the AD curve and the bisector of the represented plane intersect. This occurs, for $Y = Y_0$.

Figure 3. Effect of the interest rate on equilibrium in the goods market.



Therefore, for an interest rate i_0, an income level Y_0 is obtained. What happens if the interest rate increases? Then, the value of the AD at the origin (for Y = 0), will now be lower. Given that the slope is not affected by the interest rate, the AD curve will shift downward in parallel. The cut point with the bisector occurs at E_1, for a lower income level Y_1. Thus, the increase in the interest rate decreases the AD and, therefore, also the value of the equilibrium income.

What is now the value of the equilibrium income? Given that the $AD = \overline{A} - bi + c\,(1-t)Y$,

Y in equilibrium AD = Y

$Y = \overline{A} - bi + c\,(1-t)Y$

In this way,

$Y - c\,(1-t)Y = \overline{A} - bi$

Taking a common factor from income

$Y\,[1 - c\,(1-t)] = \overline{A} - bi$

And clearing the value of Y, we obtain the equilibrium income

$$Y^* = \frac{1}{1 - c\,(1-t)}\,(\overline{A} - bi)$$

As the spending multiplier is equal to

$$\alpha_G = \frac{1}{1 - c\,(1-t)}$$

We can express equilibrium income as follows

$$Y^* = \alpha_G(\overline{A} - bi)$$

Therefore, we can state that the equilibrium income depends negatively on the interest rate.

4.3. THE IS CURVE

4.3.1. Definition of the IS curve

The IS curve is defined as the combination of interest rates and income levels that make the goods market in equilibrium, that is, that AD = Y.

4.3.2. Graphical and analytical derivation of the IS curve

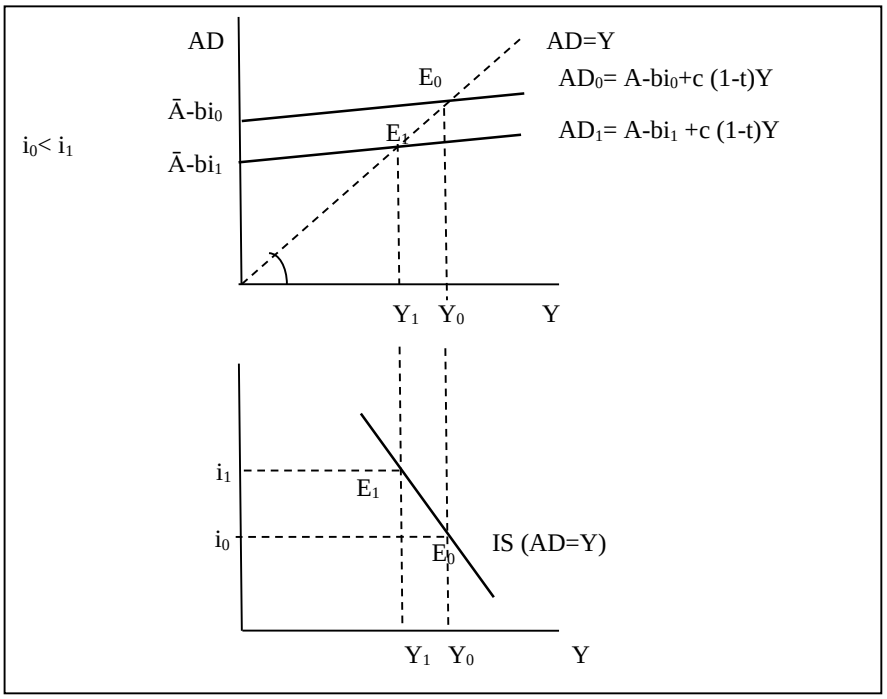
From the previous relationship that shows the equilibrium income of the goods market in a closed economy, that is, of $Y^* = \alpha_G(\overline{A} - bi)$, we can find multiple combinations of interest

rates and income levels, for which the goods market is in equilibrium. We just have to give different values to the interest rate and combine them, in each case, with the resulting income levels. Therefore, **the expression $\mathbf{Y^* = \alpha_G(\bar{A} - bi)}$ is actually the analytical expression of the IS curve.** As we will see later, this expression can be expressed in alternative ways, if the variable that is cleared is the interest rate.

How can we graphically obtain the IS curve?

To obtain the IS curve (Figure 4), we are going to relate two graphs to each other. In the upper graph we represent the equilibrium in the goods market, based on the income-spending model. In the lower graph, we are going to represent the IS curve as a combination of interest rates and income levels that make the goods market be in equilibrium, that is, AD = Y.

Figure 4. Graphical derivation of IS



If the interest rate is i_0, we can see in the upper graph that the AD is equal to the AD_0. The cut point with the bisector occurs at E_0, and an equilibrium income level equal to Y_0 is obtained. In the lower graph, we can represent this combination of interest rate (i_0) and income level Y_0. That is, we can see how for the interest rate i_0, the income that makes the AD equal to Y is Y_0. The combination of both points allows us to obtain a first point on the IS curve, that point is E_0.

To be able to represent the IS curve we need at least one other combination of interest rates and income levels that bring the goods market into equilibrium. For that, we will now suppose that the interest rate increases to i_1, obtaining AD_1. In the upper graph, we can see that the increase in the interest rate shifts the AD curve downwards. The cut point with the bisector is now E_1, which is obtained for the income level Y_1. In the lower graph, we can now represent this new combination of interest rate and income level that brings the goods market into equilibrium, that is, from (i_1, Y_1). This combination is represented in the lower graph at point E_1, which is a new point on the IS curve.

If we join the points E_0 and E_1 in the lower graph, we obtain the IS curve, which represents all the possible combinations of interest rates and income levels that make AD equal to income.

We can see that the slope of the IS curve is negative, indicating that if the interest rate increases, income must decrease to regain equilibrium in the goods market.

4.3.3. Slope and position of the IS curve

The analytical derivation of IS is equivalent to obtaining the equilibrium income when the AD is affected by interest rates, that is, it is equivalent to the analytical demonstration that we have shown of $Y^* = \alpha_G(\overline{A} - bi)$. However, as happened previously with the investment curve, the previous expression does not correspond to the IS curve represented graphically in Figure 4.

For both expressions to correspond, it is necessary to clear the interest rate from the previous function. Thus, if $Y = \alpha_G \left(\overline{A} - bi \right)$ then

$Y = \alpha_G \overline{A} - \alpha_G bi$

Therefore

$Y - \alpha_G \overline{A} = -\alpha_G bi$

Clearing the interest rate

$$i = -\frac{1}{\alpha_G b} Y + \frac{1}{b} \bar{A}$$

From this expression, the slope of the IS curve is quickly obtained by deriving interest with respect to income. The value obtained is $-\frac{1}{\alpha_G b}$. This value is negative and depends on the value of the spending multiplier and the value of b. It can be seen that, as both values increase, the slope decreases, causing the curve to become flatter. On the contrary, if these values tend to decrease, we obtain a greater slope. In the extreme case, discussed above, that b tends to zero, we obtain a totally rigid IS curve, that is, vertical. In this case, the AD does not depend on the interest rate, and therefore the equilibrium income does not change when the interest changes.

On the other hand, the change in the value of autonomous expenditure A will affect the position of the curve. Any modification of the components of A, that is, autonomous consumption, autonomous investment, public spending and transfers will affect the position of the IS curve.

Graphically, we can show how the spending multiplier and the value of b (interest rate sensitivity on investment) affect the slope of IS. Likewise, we can graphically show how changes in autonomous spending affect the position of IS. We show these cases below.

The spending multiplier and the slope of the IS curve

The spending multiplier (α_G) depends on both the tax rate and the marginal propensity to consume, as its analytical expression is $\alpha_{G=} \frac{1}{1- c\ (1-t)}$. Therefore, changes in both the marginal propensity to consume and in the tax rate will end up affecting the spending multiplier and, therefore, the slope of IS. As we know, the decrease in the tax rate and the increase in the marginal propensity to consume, lead to increases in the spending multiplier, and, as we have analytically deduced, the higher the spending multiplier, the lower the slope of IS.

Graphically we will be facing two different cases: that the tax rate changes and, therefore, that the multiplier changes and with it the slope; or, that the marginal propensity to consume changes, the multiplier changes and, therefore, the slope changes. We see both possibilities below.

a) Influence of the tax rate on the spending multiplier and on the slope of the IS curve.

In order to graphically analyze the effect of the tax rate on the IS curve, we follow the following steps:

1. We first draw an IS curve for a given tax rate

To draw an IS curve with a specific tax rate (for a value of t) we must first determine two AD curves, defined for two different interest rates and the same tax rate. These curves are represented using the income-expenditure model. For each AD curve we obtain an equilibrium income, obtaining two different combinations of interest rates and income levels, for which AD = Y. We represent these points on the plane that relates interest rates and income levels. We join both points and we have the IS curve for a given t. Next, we explain the process in detail, using Figure 5.

First, we define an AD curve. To determine this curve, we first define the value at the origin of the curve, for which it is necessary to determine an initial interest rate. Let i_0 be the initial interest rate. The value of the AD at the origin will then be equal to $\bar{A}$-bi_0. Likewise, in order to represent the AD curve we must define the value of the slope. Since we are going to analyze the change in the tax rate, we determine an initial tax rate t_0. Given that initial tax rate, the slope of the AD for that tax rate will be equal to $c(1-t_0)$. In the upper graph of Figure 5, we show the curve of AD_0 that was defined for i_0 and for t_0. The equilibrium point E_0 is obtained for income Y_0. The combination of i_0 and the income level Y_0 is represented in the lower graph of Figure 5 at point E_0, which is a point on the IS curve

In order to determine the IS curve for the tax rate t_0, we take another reference interest rate. We now assume that the interest rate is greater than, and equal to, i_1. The new AD curve (AD_1) is below the previous one, moving in parallel. The new equilibrium point is obtained at E_1, for an income level Y_1. The new combination (i_1, Y_1) can be drawn on the lower graph (point E_1). By joining the points E_0 and E_1 of the lower graph, we obtain the IS curve defined for the tax rate t_0.

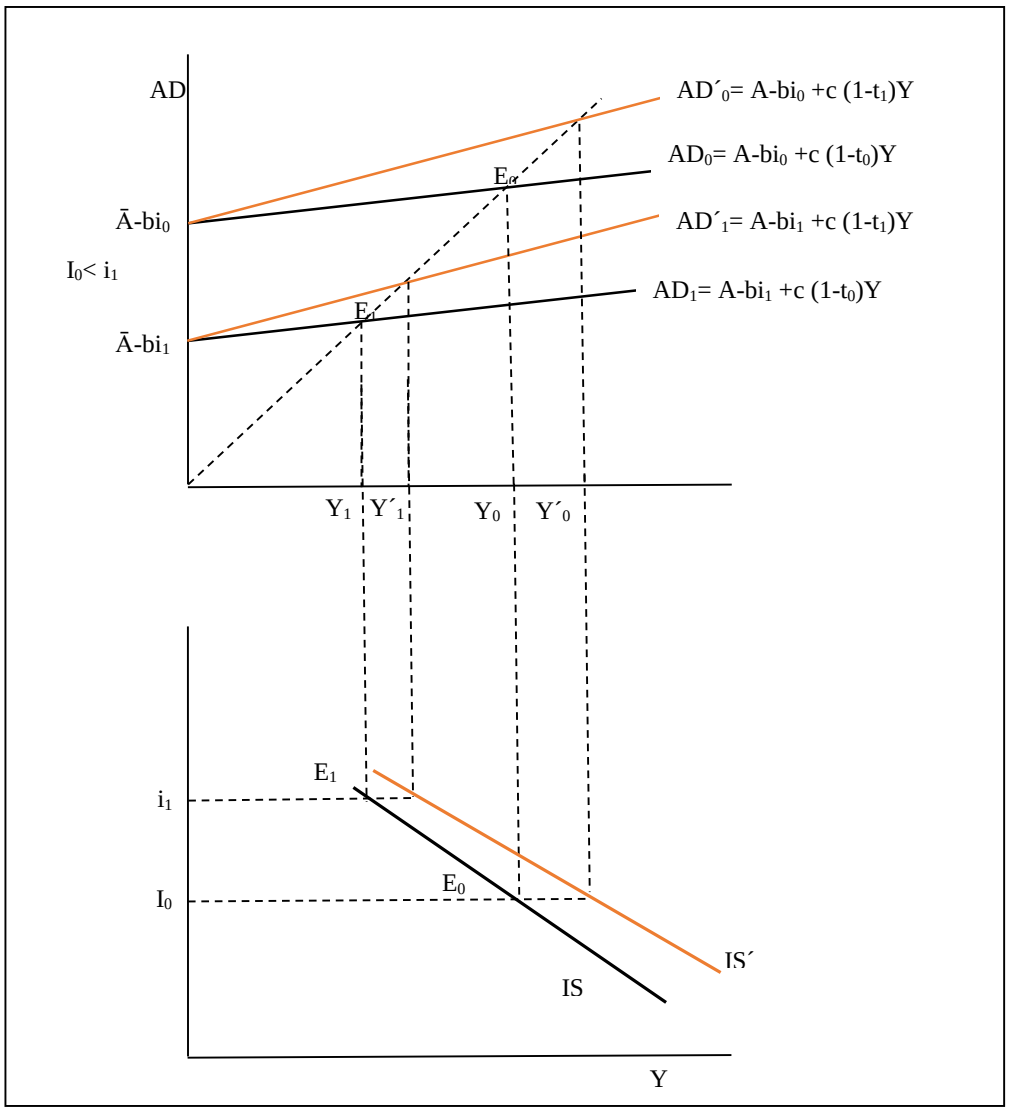
2. We change the tax rate and draw a new IS curve

Once the first IS curve has been defined, we ask ourselves: What happens if the tax rate decreases? If the tax rate decreases, the slope of the AD curve will increase, causing the spending multiplier to increase, causing equilibrium income to also increase.

To show it graphically, we assume that we start from the initial AD, AD_0 and decrease the rate to t_1. The value at the origin is unchanged, it remains $\overline{A}-bi_0$. However, as the tax rate changes, the slope of the curve will change. Specifically, as the tax rate decreases, the slope of AD_0 will increase, making the curve steeper. We obtain a new curve of AD_0 which we call AD'_0 and which we draw in red. The equilibrium point is now at E'_0 for an equilibrium income Y'_0 higher than income Y_0 (which reflects that the spending multiplier is now higher). Thus, if the tax rate decreases, the equilibrium income obtained for the interest rate i_0 is higher than the previous one, and equal to Y'_0. We can represent the equilibrium combination (i_0, Y'_0) in the lower graph, and we obtain a first point of the new IS curve.

We can repeat the same process with the curve of AD_1, that is, if we consider the curve AD_1 and decrease the tax rate to t_1, the value at the origin of said curve does not change, but the slope of the curve returns to increase by the same amount as before. We draw the new AD curve in red (parallel to the previous one in red) and denote it as AD'_1. The equilibrium point is E'_1 for an income level Y'_1. We can represent the equilibrium combination (i_1, Y'_1) in the lower graph, and we obtain the second point of the new IS curve. The new IS curve (drawn in red) is obtained by joining the points represented in the graph below. Specifically, points E'_0 and E'_1.

Figure 5. Effect of the tax rate on the IS curve



3. We compare both IS curves

If we compare the two obtained IS curves, we can see that by decreasing the tax rate and increasing the spending multiplier, the IS curve becomes flatter, shifting to the right. Therefore, the IS curve decreases its slope.

b) Influence of the marginal propensity to consume on the spending multiplier and on the slope of the IS curve.

In order to graphically analyze the effect of the marginal propensity to consume on the IS curve, we follow the same steps as before:

1. We first draw an IS curve for a given marginal propensity

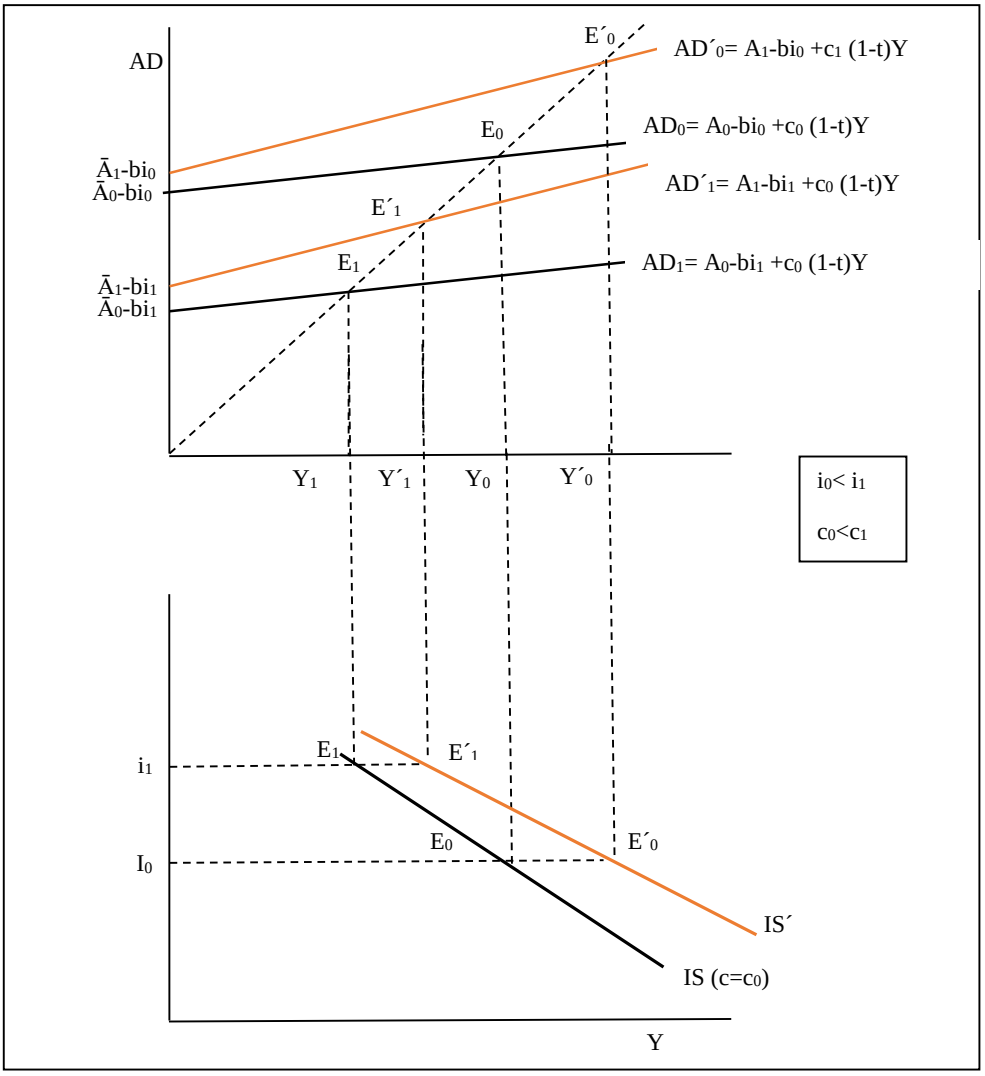
The first thing we have to know is that the marginal propensity to consume, affects both the value at the origin, and the slope of the IS curve. At the origin it affects the value of autonomous spending $\bar{A}$, then $\bar{A} = \bar{C} + \bar{I} + \bar{G} + c\overline{TR}$. The slope is also affected by c because its value is equal to c(1-t).

Therefore, we determine a value for c, which we call c_o and we draw two different ADs with that value of c. One for an interest rate i_0, and a second for a higher interest rate, i_1. The two curves are parallel. Figure 6 shows these first two curves, which we call AD_0 and AD_1, defined for i_0 and i_1, respectively. Both are defined for c_o. Given these curves, we can obtain short-run equilibrium points for the goods market. If the interest rate is i_0, the AD_0 curve intersects at E_0 with the bisector. Therefore, equilibrium occurs for income Y_0. If the interest rate is i_1, the AD_1 curve intersects the bisector at E_1. Therefore, in equilibrium, income is equal to Y_1. Thus we have two combinations of interest rates and income levels that make AD = Y. The first (i_0, Y_0) and the second, (i_1, Y_1). If we represent both combinations in the lower graph, we obtain the first IS curve.

2. We change the marginal propensity to consume and draw a new IS curve

The modification of the marginal propensity to consume has a similar double effect in the two previous AD curves. First, if the marginal propensity to consume increases, cTR increases and the value of A increases. Thus, we define A_1 as the value that autonomous spending now assumes. The AD curves shift upward. On the other hand, the slope of both AD curves increases with increasing c. Therefore, the two previous curves now have higher slopes.

Figure 6. Effect of the marginal propensity to consume on the IS curve



We draw the two new AD curves in red. The AD´$_0$ is defined for i_0 and for c_1, and the AD´$_1$ is defined for i_1 and for c_1. The points cut with the bisector now occur at E´$_0$ and E´$_1$. The obtained equilibrium income levels are Y´$_0$ y Y´$_1$, respectively. Thus, we have two new combinations of interest rates and income levels that make AD = Y. The first (i_0, Y´$_0$) and the second, (i_1, Y´$_1$). If we represent both combinations in the lower graph, we obtain the second IS curve (represented in red), for a level of marginal propensity to consume higher than the one previous.

3. We compare both IS curves

If we compare the two obtained IS curves, we can see that as the marginal propensity to consume increases and the spending multiplier increases, the IS curve becomes flatter, shifting to the right. Therefore, the IS curve decreases its slope.

Sensitivity of the investment to the interest rate and IS curve

This section shows the effect of a change in the sensitivity of the investment to the interest rate (of b) on the slope of the IS curve.

In order to graphically see how the value of b affects the IS curve, we revisit the three steps described above:

1. We first draw an IS curve for a specific value of b.

In this case, to more clearly see the effect of the value of b on IS, we are going to specify specific values for b, for A and for the interest rates i_0 and i_1. We draw the first IS curve for the following values:

$b_0=2$, $\bar{A} = 10$, $i_0= 1$, $i_1=2$

We then define two initial AD curves. The first being $AD_0= A-b_0i_0 +c(1-t)Y$. The value at the origin at $A-b_0i_0$, therefore, its value is equal to 8. Its slope is c(1-t). The second AD curve is $AD_1 = A-b_0i_1 +c(1-t)Y$. The value at the origin $A-b_0i_1= 6$. The slope, equal to the one previous, is c(1-t). We show both curves in black in the upper graph of Figure 7.

Given these curves, we can obtain short-run equilibrium points for the goods market. If the interest rate is i_0, the AD_0 curve intersects at E_0 with the bisector. Therefore, equilibrium is produced for income Y_0. If the interest rate is i_1, the AD_1 curve intersects the bisector at E_1, therefore, in equilibrium, income is equal to Y_1. Thus, we have two combinations of interest rates and income levels that make AD = Y. The first (i_0, Y_0) and the second, (i_1, Y_1). If we represent both combinations in the lower graph, we obtain the first IS curve.

2. We draw an IS curve for a higher value of b.

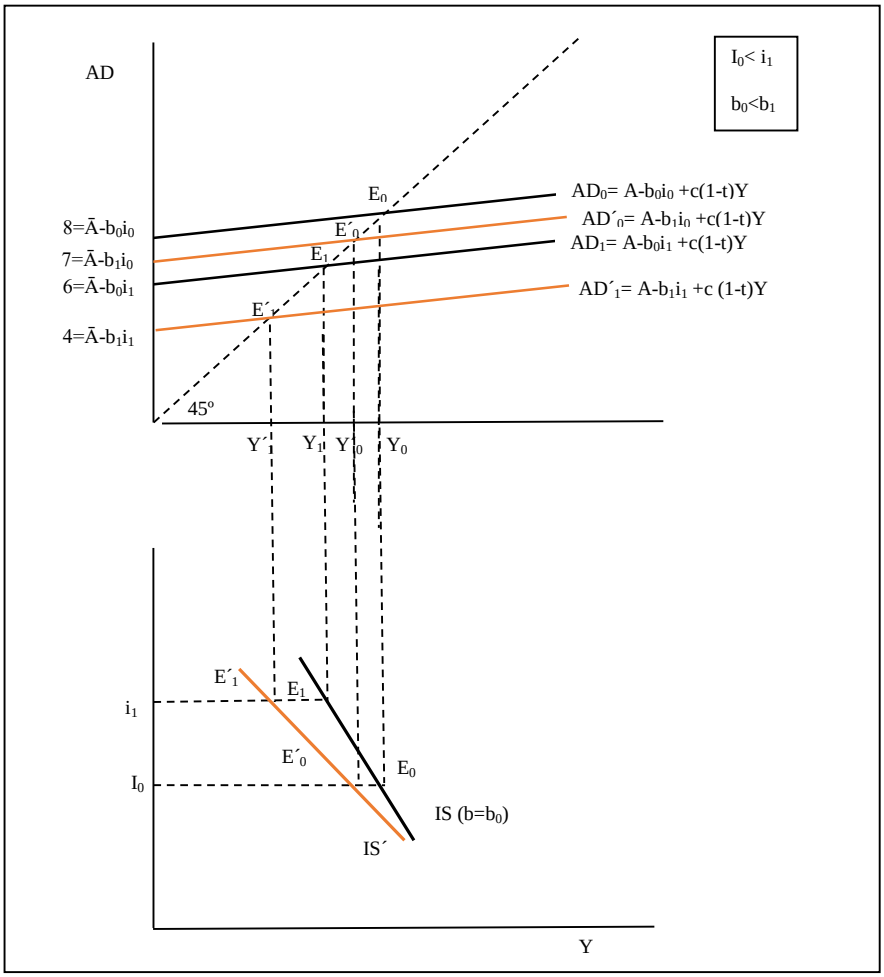
In this case, to see the effect of the value of b on IS more clearly, we are going to specify a specific higher value for b. The values of $\bar{A}$ and of the interest rates i_0 y i_1 do not change, therefore, only the value of b changes. The values are then

$b_1=3$, $\bar{A} = 10$, $i_0= 1$, $i_1=2$.

We then define two new AD curves. The first, $AD'_0= A-b_1i_0 +c(1-t)Y$. The value at the origin is $A-b_1i_0 = 7$. Its slope is c(1-t). It will thus be parallel to the previous curves. The second AD curve is $AD'_1= A-b_1i_1 +c(1-t)Y$. The value at the origin is $A-b_1i_1=4$. The slope equal to the one previous is c(1-t). We show both curves in red in the upper graph of Figure 7.

Given these curves, we can obtain short-run equilibrium points for the goods market. If the interest rate is i_0, the curve AD'_0 intersects at E'_0 with the bisector, therefore, equilibrium is produced for income Y'_0. If the interest rate is i_1, the curve AD'_1 intersects the bisector at E'_1, with the income equilibrium equal to Y'_1. Thus we have two combinations of interest rates and income levels that make AD = Y. The first (i_0, Y'_0) and the second, (i_1, Y'_1). If we represent both combinations in the lower graph, we obtain the first IS curve.

Figure 7. Effect of sensitivity of investment to the interest rate (of b) on the IS curve



3. We compare both IS curves

If we compare the two obtained IS curves, we can see that by increasing the sensitivity of the investment to the interest rate (b), the IS curve becomes flatter, shifting to the left, therefore, the IS curve decreases its slope.

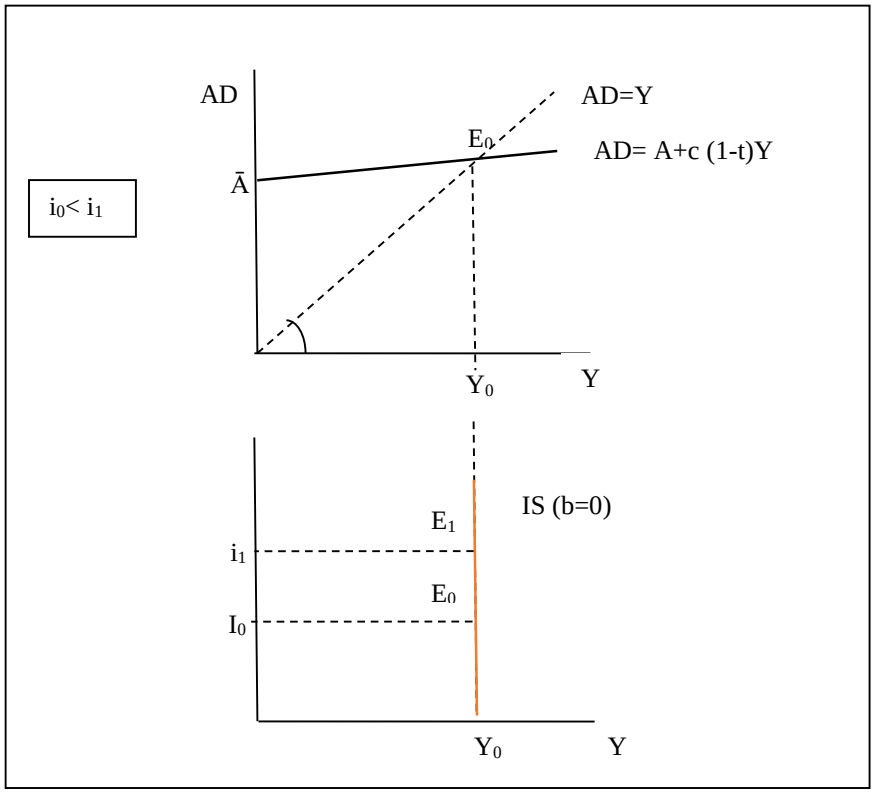
<u>Slope of IS when b = 0.</u>

A special case arises when b = 0. In this case, the investment becomes insensitive to the interest rate, therefore, the AD will not depend on the interest rate, and whatever its value, the AD will

not change. In that case, the AD curve will not change its cut with the bisector and the equilibrium income will always be the same, even if the interest rate changes.

Figure 8 indicates this situation. It can be observed that for both i_0 and i_1, the AD is the same, because said demand does not depend on the interest rate, since b=0. The IS curve is then vertical. The equilibrium income will be the same for all values of i.

Figure 8: IS curve for b=0



IS position

The position of IS depends on the value of autonomous spending $\bar{A}$. A modification of its value changes the value at the origin of the AD. Given that any modification of its components will have a modification of autonomous spending, as a consequence, and, therefore, a modification in the original value of the AD curve. Due to the interest that the modification of public spending has, we are going to assume that the modification of A is produced by a change in the same.

To see the effect of a change in public spending (G) on IS, we repeat the same steps as before:

1. We first draw an IS curve for a given autonomous spending value.

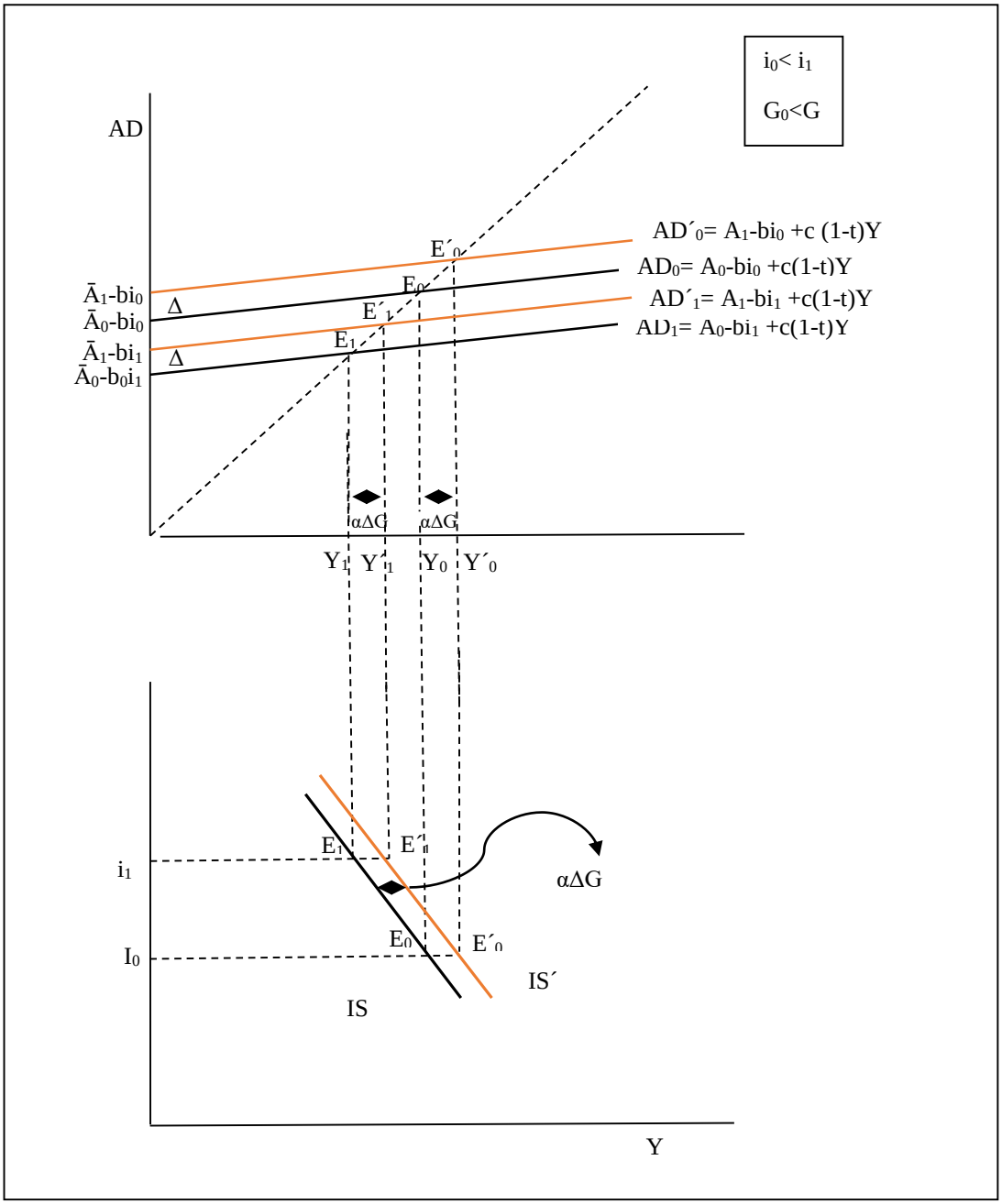
We initially assume that we have a value of G equal to G_0. This determines that the value of A is A_0. We draw two different ADs with that value of G. One for an interest rate i_o, and a second, for a higher interest rate, i_1. The two curves are parallel. Figure 9 shows these first two curves, which we call AD_0 and AD_1, defined respectively, for i_0 and i_1. Both are defined for G_o and, therefore, for A_0. Given these curves, we can obtain short-run equilibrium points for the goods market. If the interest rate is i_0, the AD_0 curve intersects at E_0 with the bisector, therefore, equilibrium is produced for income Y_0. If the interest rate is i_1, the AD_1 curve intersects the bisector at E_1, with the income equilibrium equal to Y_1. Thus, we have two combinations of interest rates and income levels that make AD = Y. The first (i_0, Y_0) and the second, (i_1, Y_1). If we represent both combinations in the lower graph, we obtain the first IS curve.

2. We increase the value of public spending and draw a new IS curve.

If we increase public spending, the two previously drawn AD curves (black) shift upward by an amount equal to the increase in spending (red curves). Since the change in G does not affect the slope of the AD, the upward shift is parallel.

Given these curves, we can obtain short-run equilibrium points for the goods market. If the interest rate is i_0, the AD'_0 curve intersects at E'_0 with the bisector, producing the income equilibrium at Y'_0. If the interest rate is i_1, the AD'_1 curve intersects the bisector at E'_1, therefore, the income equilibrium is equal to Y'_1. Thus we have two combinations of interest rates and income levels that make AD = Y. The first (i_0, Y'_0) and the second, (i_1, Y'_1). If we represent both combinations in the lower graph, we obtain the first IS curve.

Figure 9. Effect of public spending on the IS curve



3. We compare both IS curves

If we compare the two obtained IS curves, we can observe that, as public spending increases, the IS curve shifts to the right. Given the increase in public spending is ΔG, the increase in income due to this increase in spending is αΔG. Therefore, the IS curve shifts to the right in parallel at that value. Thus, a fiscal policy consisting of an increase in public spending affects the IS curve, by shifting it to the right, by αΔG. The shift to the right will be greater as public spending increases, but also as the value of the spending multiplier increases.

If autonomous consumption or investment were to increase, the effect on the IS curve would be similar: a shift of IS to the right by a value equal to $\alpha\Delta C$ and $\alpha\Delta I$, respectively, where ΔC is the increase in consumption, and ΔI is the increase in investment. Finally, if the increase were in transfers, the IS curve would also shift to the right, but the value of the shift would be $\alpha c\Delta TR$.