THE INCOME-EXPENDITURE MODEL: THE PUBLIC SECTOR AND EQUILIBRIUM IN A CLOSED ECONOMY

THEME 3

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3.1. THE ROLE OF CONSUMPTION IN A CLOSED ECONOMY

In a closed (three-sector) economy, in addition to households and companies, the public sector and financial agents also participate. Thus, the disposable income of households (YD) is no longer equivalent to the income of the economy (Y).

Basically, the introduction of the public sector in the macroeconomic model affects DA through three components: public expenditure (G), public transfers (TR) and public revenues (mainly tax revenues). The determination of these three components is known as fiscal policy. If we simplify, we can say that fiscal policy consists of the determination by the public sector (through legal procedures) of the value of these three variables.

On the one hand, the public sector determines the value of public spending and transfers when public budgets are approved. We express the determination of these values by marking the variables with a dash above them. Thus,

$$G = \overline{G} \text{ y } TR = \overline{TR}$$

On the other hand, the definition of tax policy also involves determining the value of the tax rates applied to income or other taxable events. For the sake of simplicity, let us assume that there is a single tax levied on income Y. The public sector, in determining its tax policy, decides the tax rate or rates levied on income. Again, for simplicity, we assume that there is a single tax rate (t) levied on income Y. The tax revenue (government revenue) will be T = tY. Since the Public Sector cannot take in more money than the existing income, t is a value between zero and one. Thus,

$$T = tY, \text{ where } 0 < t < 1$$

The introduction of the public sector in the economic model implies that income is no longer equal to disposable income. Household disposable income now increases with the transfers received from the public sector, but in turn decreases with the taxes they have to pay. Therefore:

$$YD = Y + \overline{TR} - tY = \overline{TR} + (1-t)\,Y$$

How does the determination of fiscal policy affect the consumption function?

Since Y is not equal to YD and consumption depends on disposable income, when the public sector is introduced in our model, the consumption function is no longer the one we explained in the previous topic. Now the consumption function is expressed as follows:

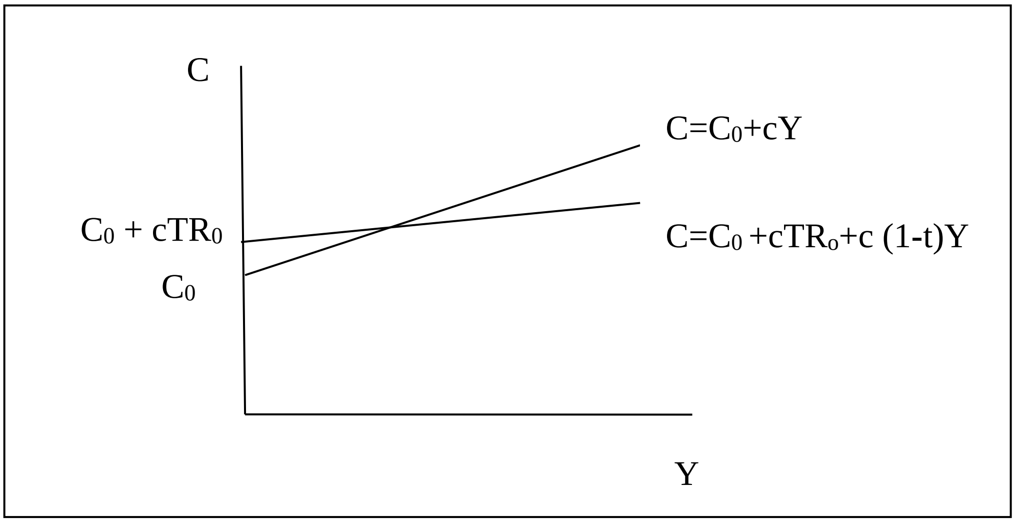
$$C = \overline{C} + c(Y + \overline{TR} - tY) = \overline{C} + c\overline{TR} + c(1-t)Y$$

Transfers raise autonomous consumption expenditure by an amount equal to $c\overline{TR}$. The part of consumption that does not depend on income is no longer just autonomous consumption $\overline{C}$ but is increased by $c\overline{TR}$.

On the other hand, taxes reduce consumption, since they reduce disposable income. Thus, households deduct tax payments from their income (Y), and on the remaining income they make their decisions to save or consume. Thus, now if income increases by one unit, the increase in consumption will be equal to c(1-t) monetary units, this value indicating the slope of the new consumption function. Since (1-t) is less than unity, the value of c(1-t) is less than c.

Two main differences are then observed between the consumption function in a two-sector and a three-sector economy. First, the introduction of the public sector increases the part of consumption that does not depend on income and shifts the consumption curve upwards. Second, the introduction of the public sector modifies the slope of the curve making it smoother (Figure 1).

Figure 1. Comparison of the consumption function with two and three sectors



3.2. AGGREGATE DEMAND IN A CLOSED ECONOMY

In a closed economy, aggregate demand is the sum of consumption, investment and public spending.

$$DA = C + I + G$$

Consumption in the closed economy is given by

$$C = \overline{C} + c(Y + \overline{TR} - tY) = \overline{C} + c\overline{TR} + c(1-t)Y$$

Investment, as before, is constant, and public spending is defined by fiscal policy. Therefore,

I = $\overline{I}$ y G = $\overline{G}$

In this way:

$$DA = \overline{C} + c\overline{TR} + \overline{I} + \overline{G} + c\ (1-t)\ Y = \overline{A} + c\ (1-t)Y$$

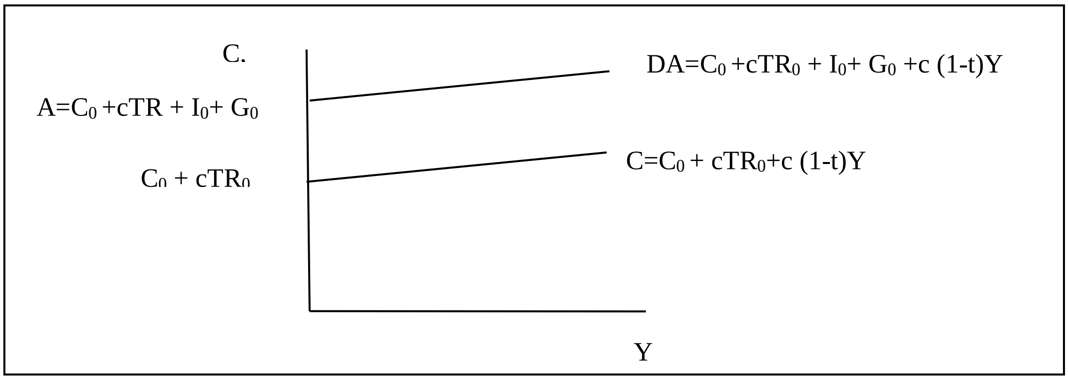
Now the autonomous expense $\overline{A}$ is the same as

$$\overline{A} = \overline{C} + c\overline{TR} + \overline{I} + \overline{G}$$

The DA in a simple economy is, therefore, a simplification or special case of the DA of the closed economy. So, we can say that the DA of a simple economy is equivalent to that of a closed economy, where the TR, the G, and the tax rate t are zero.

Graphically (Figure 2), the DA can be represented from the consumption function, to which we add the autonomous components $c\overline{TR}$, $\overline{I}$ and $\overline{G}$. It can be seen that the DA has the same slope as the consumption function (parallel) but is located above it by an amount equal to the sum of G+I.

Figure 2: DA in a closed economy



3.3. EQUILIBRIUM IN A CLOSED ECONOMY

As we stated above, the equilibrium of the economy occurs where income is equal to DA. Thus, given that in equilibrium

$$DA = Y, \text{ y la } DA = \bar{A} + c(1 - t)Y$$

Therefore $Y = \bar{A} + c(1 - t)Y$

So, $Y - c(1 - t)Y = \bar{A}$. And $Y[1- c(1 - t)] = \bar{A}$

By clearing $Y^* = \bar{A}/[1- c(1 - t)]$

The term $1/[1- c(1 - t)]$ is called the expenditure multiplier, since its value is always greater than 1 (since both c and t are positive values less than unity). The notation α_G can be used to designate its value.

$$\text{So, if } \alpha_G = 1/[1- c(1 - t)], \quad Y^* = \alpha_G \bar{A}$$

Therefore, the equilibrium income depends on the autonomous expenditure, and on the value of the multiplier, which in turn depends on the marginal propensity to consume and the tax rate.

3.4. THE MULTIPLIER IN A CLOSED ECONOMY

The expenditure multiplier in a closed economy reflects the increase in income in the face of a unit increase in autonomous expenditure $\bar{A}$.

Given that $Y^* = \alpha_G \bar{A}$, y $\alpha_G = 1/[1- c(1 - t)]$

an increase in $\bar{A}$ ($\Delta \bar{A}$), increases $\alpha_G \bar{A}$ ($\alpha_G \Delta \bar{A}$), which translates into an increase in Y^* (ΔY^*). By what value?

If $\Delta Y^* = \alpha_G \Delta\bar{A}$

then

$$\Delta Y^* / \Delta\bar{A} = \alpha_G$$

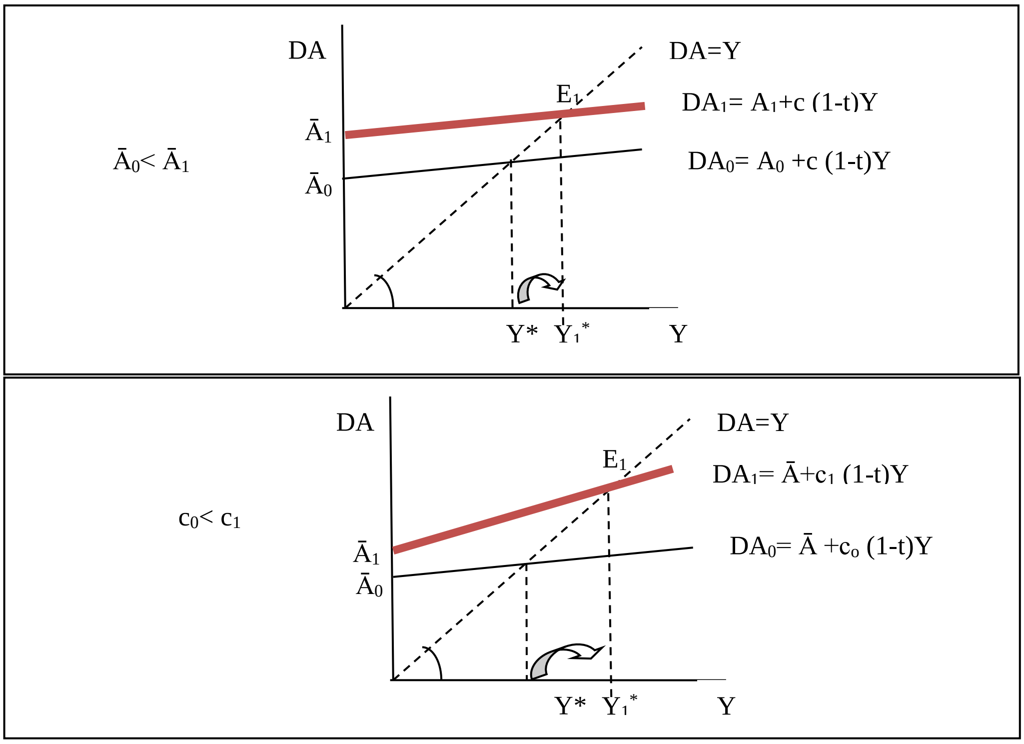
Therefore, α_G reflects the increase in equilibrium income in the face of increases in autonomous expenditure, this being the expenditure multiplier in a closed economy. At this point it is worth remembering that $\bar{A}$ depends on $\bar{C}$, $c\overline{TR}$, $\bar{I}$ and $\bar{G}$. Therefore, if any of these variables increases, equilibrium income increases. It should also be noted that if α_G increases (due to an increase in c or a decrease in t), equilibrium income increases.

The upper and lower graphs in Figure 3 reflect respectively the effect of the increase in equilibrium income in the presence of increases in $\bar{A}$ and α_G in a closed economy. In both graphs we start from an initial equilibrium position Y*. That value is obtained at the cut-off point of the initial DA (DA_0) and the bisector or straight line that equals DA and Y.

In the first case (top graph), a larger value of $\bar{A}$, shifts upward and parallel the DA curve to DA_1. The new DA_1 curve (red) now cuts the bisector at E_1. That point is obtained for income level Y_1^*. A higher-income value than the initial one (Y*). Thus, higher values of $\bar{A}$ lead to higher equilibrium income values.

In the second case (lower graph), a higher value of the expenditure multiplier (in the graph it is reflected by the increase of c), causes the position of the demand curve at the origin to change (since now $\bar{A}$ is higher, as $\bar{A} = \bar{C} + c\overline{TR} + \bar{I} + \bar{G}$), and also causes the new DA curve (DA_1 highlighted in red) to have a steeper slope. The change in the curve causes it to intersect the bisector at point E_1, resulting in an equilibrium income Y_1^* higher than the initial value Y*.

Figure 3. Equilibrium income. Effect of the increase in autonomous expenditure and the expenditure multiplier



3.5. EFFECTS OF A CHANGE IN FISCAL POLICY ON EQUILIBRIUM INCOME IN A CLOSED ECONOMY

The change in fiscal policy can be affected by changes in G, TR and the tax rate t. We say that, if G and/or TR increase, an expansionary fiscal policy is being carried out, while, if these values decrease, the fiscal policy will be restrictive. Likewise, a decrease in the tax rate is an expansionary fiscal policy, and an increase in the tax rate is a restrictive fiscal policy.

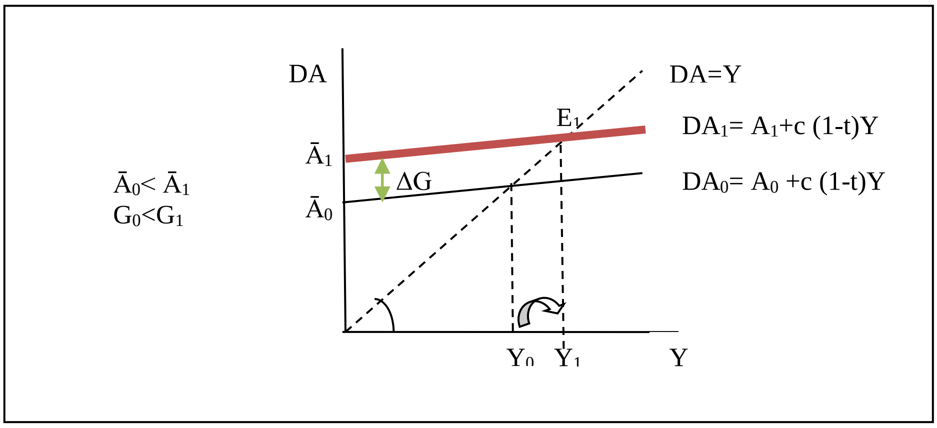
Below, we show graphically and analytically the effects of an increase in G and TR and a decrease in the tax rate.

3.5.1. Effects of Public Spending on Goods and Services

An increase in government purchases (G) implies a change in autonomous expenditure $\bar{A}$, and, therefore, in DA. That increase then generates a disequilibrium between DA and Y (DA>Y) that will eventually grow the economy's income.

Graphically, Figure 4 shows the effect of the increase in public spending. We start from an initial equilibrium position between DA_0 and the economy's income. This equilibrium point is obtained for income Y_0. DA_0 is defined for a level of public expenditure equal to G_0. From that position, we assume that public expenditure increases up to G_1, so that $G_1 > G_0$. That is, an expansionary fiscal policy of increased public spending is carried out. The increase in G causes an increase in autonomous spending ($\bar{A}$), and the DA curve shifts upward at the origin by a value equal to the increase in G. Since the slope of the DA curve is not affected by this change, the upward shift of DA is parallel. Now, at Y_0, DA is higher than income, so income will start to increase until it reaches a new equilibrium point, which occurs at E_1. This equilibrium happens for income level Y_1. As can be seen, the final effect of the increase in public spending is the increase in income from Y_0 to Y_1.

Figure 4: Effect of increased government spending on equilibrium income



What is the value of the equilibrium rent increase in the face of increased government spending? We can derive the value of the increase in income analytically as follows:

We want to know how much ΔY is worth, i.e., the difference between Y_1 and Y_0. Graphically, we can observe that Y_0 equals DA_0 at point Y_0 and that $Y_1 = DA_1$ at Y_1. Therefore:

$$\Delta Y = Y_1 - Y_0 = DA_1(Y_1) - DA_0(Y_0)$$

Given that

$$DA_0\,(Y_0) = \bar{C} + c\,\overline{TR} + \bar{I} + G_0 + c\,(1-t)\,Y_0$$

$$DA_1\,(Y_1) = \bar{C} + c\,\overline{TR} + \bar{I} + G_1 + c\,(1-t)\,Y_1$$

Then

$$\Delta Y = DA_1(Y_1) - DA_0\,(Y_0) = \bar{C} + c\,\overline{TR} + \bar{I} + G_1 + c\,(1-t)\,Y_1 - [\bar{C} + cTR_0 + \bar{I} + \bar{G} + c\,(1-t)\,Y_0]$$

Simplifying

$\Delta Y = G_1 + c\ (1\text{-}t)\ Y_1 - G_0 - c\ (1\text{-}t)\ Y_0$

Rearranging and taking out common factor

$\Delta Y = G_1 - G_0 + c\ (1\text{-}t)\ Y_1 - c\ (1\text{-}t)\ Y_0 = G_1 - G_0 + c\ (1\text{-}t)\ (Y_1 - Y_0) = \Delta G + c\ (1\text{-}t)\ \Delta Y$

So

$$\Delta Y = \Delta G + c\ (1\text{-}t)\ \Delta Y$$

In that way

$\Delta Y - c\ (1\text{-}t)\ \Delta Y = \Delta G$

$\Delta Y[1 - c\ (1\text{-}t)] = \Delta G$

Therefore, the increase in income generated by an increase in public expenditure is

$$\Delta Y = 1/\ [1 - c\ (1\text{-}t)]\Delta G$$

since

$$1/\ [1 - c\ (1\text{-}t)] = \alpha_G$$

$$\Delta Y = \alpha_G\ \Delta G$$

In other words, the increase in income is equal to the increase in public spending multiplied by the spending multiplier.

There are several conclusions to be drawn:

1. An increase in public spending positively affects the increase in income. That is, an increase in public spending increases income. Likewise, it follows from the previous expression that a decrease in public spending decreases the economy's income (it is important to remember that we are under the assumption of a closed economy in the short run).

2. The above expression also indicates that the increase in government spending results in an increase in income that is larger than the initial increase in government spending. This is because α_G is positive and greater than unity.

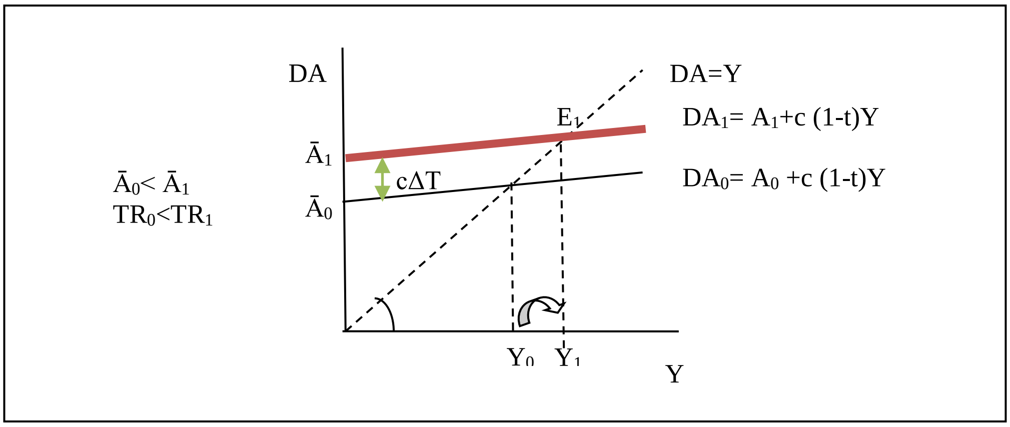
3. Both the marginal propensity to consume and the tax rate affect the multiplier and, therefore, can increase or attenuate the positive effect of public spending on income. If the marginal propensity to consume increases and/or the tax rate decreases, the multiplier (α_G) increases in value and, therefore, the effect of increased public spending on income increases.

3.5.2. Effects of Public Transfers

An increase in public transfers (TR) implies a change in autonomous expenditure $\bar{A}$ and therefore in DA. This increase then generates an imbalance between DA and Y (DA>Y) that will eventually increase the economy's income.

Graphically, Figure 5 shows the effect of increasing public transfers. We start from an initial equilibrium position between DA_0 and the economy's income. This equilibrium point is obtained for income Y_0. DA_0 is defined for a level of transfers equal to TR_0. From that position, we assume that public transfers increase up to TR_1, so that $TR_1 > TR_0$. In other words, an expansionary fiscal policy of increasing transfers is carried out. The increase in TR causes an increase in autonomous expenditure ($\bar{A}$), shifting the DA curve upward at the origin by a value equivalent to the increase in TR multiplied by the marginal propensity to consume (it is worth remembering that TR are part of $\bar{A}$ but not in their total value, but in the part of the transfers that are devoted to consumption, i.e., in cTR). Since the slope of the DA curve is not affected by this change, the upward shift of DA is parallel. Now, at Y_0, DA is higher than income, so income will start to increase until a new equilibrium point is reached, which occurs at E_1. This equilibrium is reached for income level Y_1. As can be seen, the final effect of the increase in TR is the increase in income from Y_0 to Y_1.

Figure 5: Effect of the increase in public transfers on equilibrium income



What is the value of the equilibrium rent increase in the face of increased public transfers? We can derive the value of the increase in income analytically as follows:

We want to know how much ΔY is worth, i.e., the difference between Y_1 and Y_0. Graphically, we can observe that Y_0 is equal to DA_0 at point Y_0, and that $Y_1 = DA_1$ at Y_1. Therefore

$\Delta Y = Y_1 - Y_0 = DA_1(Y_1) - DA_0(Y_0)$

Given that

$DA_0(Y_0) = \overline{C} + cTR_0 + \overline{I} + \overline{G} + c(1-t)Y_0$

$DA_1(Y_1) = \overline{C} + cTR_1 + \overline{I} + \overline{G} + c(1-t)Y_1$

Then

$\Delta Y = DA_1(Y_1) - DA_0(Y_0) = \overline{C} + cTR_1 + \overline{I} + \overline{G} + c(1-t)Y_1 - [\overline{C} + cTR_0 + \overline{I} + \overline{G} + c(1-t)Y_0]$

Simplifying

$$\Delta Y = cTR_1 + c(1-t)Y_1 - cTR_0 - c(1-t)Y_0$$

Rearranging and taking out common factor

$\Delta Y = cTR_1 - cTR_0 + c(1-t)Y_1 - c(1-t)Y_0 = c(TR_1 - TR_0) + c(1-t)(Y_1 - Y_0) = c\Delta TR + c(1-t)\Delta Y$

So

$\Delta Y = c\Delta TR + c(1-t)\Delta Y$

In that way

$\Delta Y - c(1-t)\Delta Y = c\Delta TR$

$\Delta Y[1 - c(1-t)] = c\Delta TR$

Therefore, the increase in income generated by an increase in TR is

$$\Delta Y = 1/[1 - c(1-t)]c\Delta TR$$

since

$$1/[1 - c(1-t)] = \alpha_G$$

$$\Delta Y = \alpha_G c\Delta TR$$

In other words, the increase in income is equal to the increase in transfers multiplied by the expenditure multiplier and by the marginal propensity to consume.

There are several conclusions to be drawn:

1. An increase in TR positively affects the increase in income. Likewise, it follows from the above expression that a decrease in TR decreases the economy's income.

2. The amount by which the rent increases depends on several factors. First, the amount of the increase in TRs. Second, the marginal propensity to consume. The higher its value, the greater the effect of transfers on income. Finally, the higher the expenditure multiplier, the greater the effect of the increase in transfers on income. It should be remembered that the multiplier depends on the marginal propensity to consume and tax rates.

3. Finally, it is worth comparing the effect on income of an equal increase in public spending and transfers. We have seen that the increase in income due to the increase in G is equal to:

$$\Delta Y_1 = \alpha_G \Delta G.$$

In addition, we have found that the increase in income due to the increase of TR is

$\Delta Y_2 = \alpha_G c\Delta TR$.

Since c is a positive number but less than unity, if $\Delta G = \Delta TR$

Therefore,

$\Delta Y_1 > \Delta Y_2$

In other words, an increase in G has a greater effect on income than an increase of the same amount in TR.

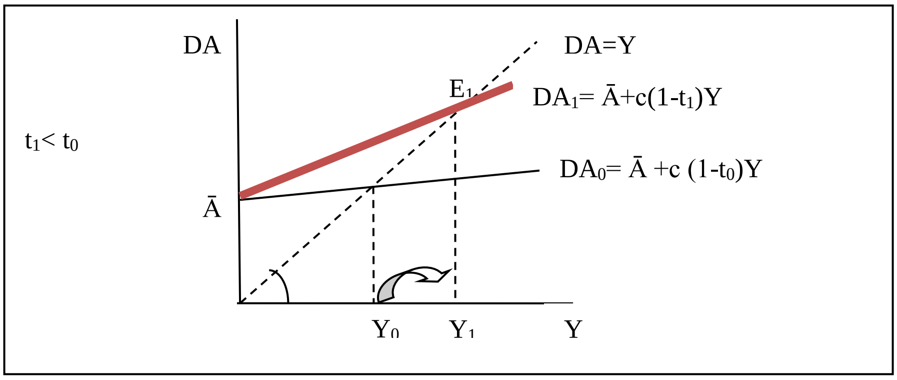
Why does this happen? If the public sector increases G, all the increase in spending becomes an increase in DA. However, when the TR increases, for example, due to an increase in pensions, the individuals who receive these transfers devote a part to consumption (which depends on the marginal propensity to consume), and another part to savings (i.e., they do not convert them into expenditure). Thus, only a portion of the transfers is converted into expenditure (in greater proportion). Specifically, only a value equivalent to cTR is converted into DA. For this reason, the effect of an increase in G on income is greater than the effect of an increase in TR.

3.5.3. Effects of the Tax Rate

A decrease in the tax rate (t) means that households have to pay less taxes and, therefore, have more disposable income to spend, causing DA to increase. This increase then generates an imbalance between DA and Y (DA>Y) which will eventually increase the economy's income.

Graphically, Figure 6 shows the effect of the decrease in the tax rate. We start from an initial equilibrium position between DA_0 and the economy's income. This equilibrium point is obtained for income Y_0. DA_0 is defined for a tax rate equal to t_0. From that position, we assume that the tax rate decreases up to t_1, so that $t_0 > t_1$. That is, an expansionary fiscal policy of decreasing taxes is carried out. The decrease in t causes a change in the slope of the DA curve. Specifically, a decrease in t increases the slope of the DA curve. Since tax rates do not affect autonomous spending ($\bar{A}$), the position at the origin of the DA curve is not changed. The new tax rate affects only the slope of the curve, causing it to become steeper. Now, at Y_0, DA is higher than income, so income will start to increase until it reaches a new equilibrium point, which occurs at E_1. This equilibrium occurs for income level Y_1. As can be seen, the final effect of the decrease in t is the increase in income from Y_0 to Y_1.

Figure 6: Effect of a decrease in the tax rate on equilibrium income



What is the value of the increase in equilibrium income in the face of the decrease in the tax rate? We can derive the value of the increase in income analytically as follows:

We want to know how much ΔY is worth, i.e., the difference between Y_1 and Y_0. Graphically, we can observe that $Y_0 = DA_0$ at point Y_0 and that $Y_1 = DA_1$ at Y_1. Therefore

$$\Delta Y = Y_1 - Y_0 = DA_1(Y_1) - DA_0(Y_0)$$

Given that

$$DA_0\,(Y_0) = \overline{C} + c\overline{TR} + \overline{I} + \overline{G} + c(1-t_0)\,Y_0$$

$$DA_1\,(Y_1) = \overline{C} + c\overline{TR} + \overline{I} + \overline{G} + c(1-t_1)\,Y_1$$

Then

$$\Delta Y = DA_1(Y_1) - DA_0\,(Y_0) = \overline{C} + c\overline{TR} + \overline{I} + \overline{G} + c(1-t_1)\,Y_1 - [\overline{C} + c\overline{TR} + \overline{I} + \overline{G} + c(1-t_0)\,Y_0]$$

Simplifying

$$\Delta Y = c\,(1-t_1)\,Y_1 - c\,(1-t_0)\,Y_0$$

We clear the parentheses

$$\Delta Y = cY_1 - ct_1Y_1 - c\,Y_0 + ct_0Y_0 \text{ (equation 1)}$$

We know that $\Delta Y = Y_1 - Y_0$

Therefore, $Y_1 = \Delta Y + Y_0$

We can then substitute the value of Y_1 into equation 1

$$\Delta Y = cY_1 - ct_1Y_1 - c\,Y_0 + ct_0Y_0 = c\,(\Delta Y + Y_0) - ct_1(\Delta Y + Y_0) - cY_0 + ct_0Y_0$$

Again, we clear the parentheses

$$\Delta Y = c\Delta Y + cY_0 - ct_1\Delta Y - ct_1Y_0 - cY_0 + ct_0Y_0$$

we neutralize cY_0

$$\Delta Y = c\Delta Y - ct_1\Delta Y - ct_1Y_0 + ct_0Y_0$$

We draw common factor, on the one hand, from ΔY and on the other from Y_0. So that

$$\Delta Y = c(1 - t_1)\Delta Y - c(t_1 - t_0)Y_0 = c(1 - t_1)\Delta Y - c\Delta tY_0$$

Therefore,

$$\Delta Y = c(1 - t_1)\Delta Y - cY_0\Delta t$$

In that way

$\Delta Y - c(1 - t_1)\Delta Y = - cY_0\Delta t$

We draw common factor *again*

$\Delta Y[1 - c(1 - t_1)] = - cY_0\Delta t$

We clear ΔY

$$\Delta Y = -\frac{cY_0\Delta t}{\left[1-c(1-t_1)\right]}$$

In other words, a decrease in the tax rate generates an increase in income. There are several conclusions to be drawn:

1. The negative sign of the above relationship indicates that the relationship between the Δt and ΔY is negative. Therefore, if the tax rate increases, income decreases, and if the tax rate decreases income increases.

2. The amount by which rent varies depends on several factors. First, on the value of the variation in t (Δt). The greater the variation in the tax rate, the greater the effect on income. Second, it depends on Y_0. If the initial income of the country is higher, the effect of the tax rate on income will also be higher. In other words, the effect of the tax rate on income depends on the initial level of the country's income. The richer the country, the greater the effect of a change in the tax rate. Finally, thirdly, it also depends on the final tax rate remaining in the economy (t_1). The higher the final tax rate in the economy, the smaller the effect of the change in the final tax rate on income. Therefore, even if the change in the tax rate is of the same amount (for example, Δt =-0.2), the effect of this change will depend on the final value of the tax rate (the smaller the final tax rate, the greater the effect). Thus, even if the variation is equal, a change from 0.3 to 0.1 will have a greater effect than a change from 0.9 to 0.7 (although in both cases Δt =-0.2, in the second case t_1 =0.7, which is greater than =0.1).

3.6. THE BUDGET BALANCE. EFFECTS OF A CHANGE IN FISCAL POLICY ON THE BUDGET BALANCE

The budget balance is the difference between the government's revenues and its total expenditures. The government's revenues are mainly tax revenues, although it also receives revenues from other items such as fees and property income. The State's expenditures are also varied and can be ranked according to different criteria.

If we simplify, we can say that government revenues are tax revenues T and total expenditures are made up of public expenditures G and public transfers TR.

We have seen above that when the public sector determines its fiscal policy it determines a value for G, for TR and determines a tax rate t, which given the level of income of the economy determines the value of tax revenues T. Therefore, we can define the budget balance (SP) as the difference between T and the sum of G + TR, once the values of G, TR and t have been defined by the public sector.

The SP is then defined as follows:

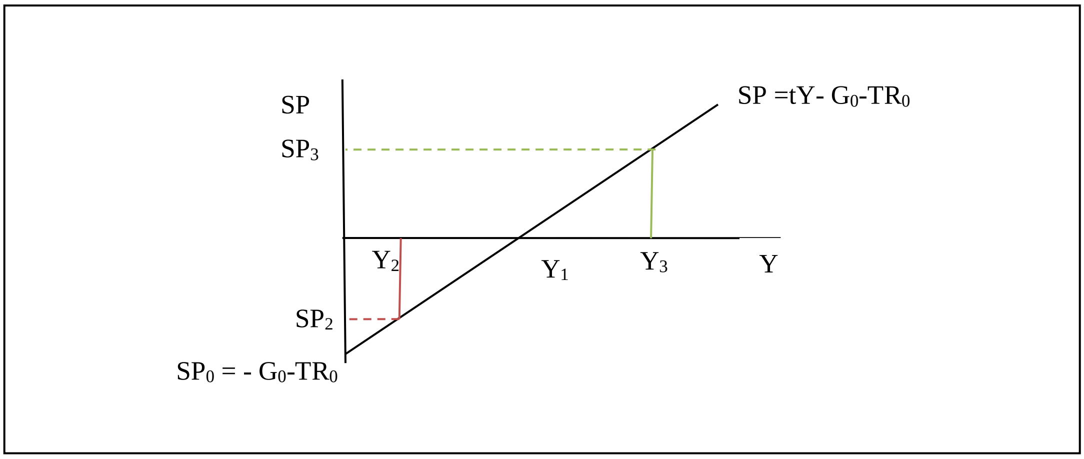
$$SP = T\text{-}(G+TR) = tY\text{-} (\overline{G} + \overline{TR})$$

The SP can be positive if $tY > (\overline{G} + \overline{TR})$. In this case there is a budget surplus. May be negative if $tY < (\overline{G} + \overline{TR})$, in which case there is a government deficit. Or, it can be zero, when total revenues and total expenditures are equal, i.e., if $tY = (\overline{G} + \overline{TR})$. In the latter case, the budget is balanced.

The final value of the SP depends on the fiscal policy established by the public sector, but it also depends on the income level of the economy. We can see graphically how, given the same fiscal policy, the value of income finally determines whether we are in a situation of budget deficit, surplus or balance. Figure 7 shows the relationship between SP and revenue. The SP is represented on the ordinate axis and the income on the abscissa axis.

When income is 0, there is no tax revenue and, therefore, all government spending equals a deficit or negative SP. In this case, the $SP = -(\overline{G} + \overline{TR})$. As income begins to increase, tax funds begin to be collected, and since expenditures are fixed, the deficit begins to decrease. For a level of income Y_1, we can observe that SP=0, since revenues $tY_1 = G_0 + TR_0$, so there is budget balance for that level of income. If income continues to grow, then budget revenues will be greater than total expenditures and we will be in a surplus situation. In general, given the fiscal policy, i.e., given the value of t, G and TR, income determines whether there is a budget deficit, balance or surplus. In Figure 7, for any value below Y_1 there is a budget deficit, while for any value above Y_1 there is a surplus. Thus, for example, if income is Y_2, we can see graphically that there is a deficit equal to SP_2. Whereas, if income is Y_3, there is a positive SP equal to SP_3. Thus, the public deficit depends not only on fiscal policy but also on the value of the economy's income. Therefore, deficits are more likely to occur in times of recession and surpluses in times of economic expansion.

Figure 7. The budget balance

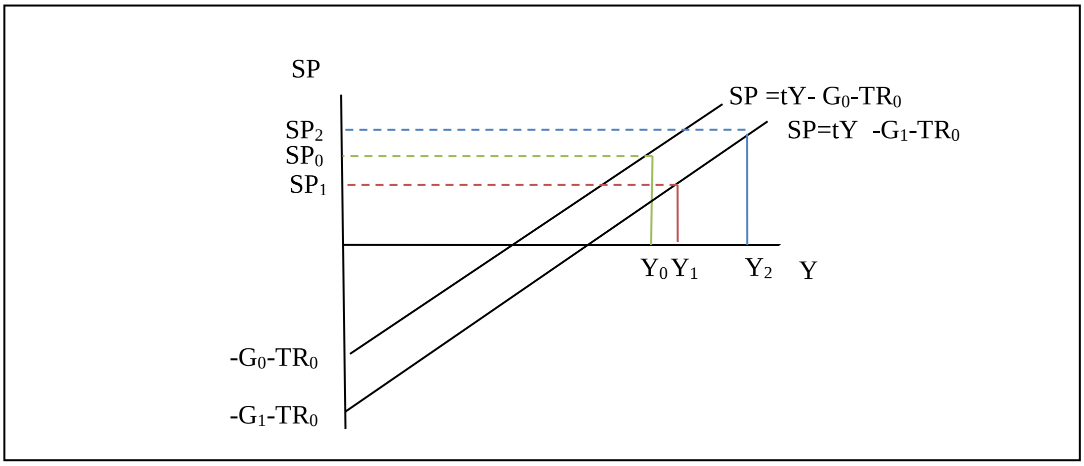


3.6.1. Effects of Public Spending on Goods and Services

An increase in public spending, i.e., in G, has a direct effect on the budget balance, causing it to decrease in value. However, an increase in public spending also has a positive effect on income (we have seen above that if G increases, there is an increase in income equal to $\Delta Y_1 = \alpha_g \, \Delta G$). If income increases, then we will have higher tax revenues and consequently the budget balance will improve. We then have an opposite effect. On the one hand, if G increases, S worsens, but if G increases, Y increases, tY increases and SP improves. Which of the two effects is greater?

We try to answer this question graphically. Figure 8 shows the effect of an increase in G on SP.

Figure 8. Effect of an increase in G on SP.



We start from an initial situation of positive SP or surplus, given the initial fiscal policy with a given level of TR, t and the level of G = G_0. Likewise, we start from an initial situation of income Y equal to Y_0. We can observe that, for that level of income, SP = SP_0. What happens if G increases? If G increases to G_1, the value at the origin of public deficit will now be higher, up to -G_1-TR. We must then draw its value below the previous one. Since the tax rate has not changed, the slope of the SP curve does not change, and the curve is then shifted downward in a parallel pattern.

What is the value of SP now? The level of income is no longer valid, since the increase in G causes income to increase. Let us imagine that income increases up to Y_1. Then, given the new fiscal policy (new value of G), SP is now SP_1 lower than before. We could conclude that if G increases, SP decreases. However, let us imagine that income increases up to Y_2. Then, with the new fiscal policy SP is now SP_2, higher than SP_0. We could conclude that, if G increases, SP increases. So, what is the final effect? It will be negative if income increases relatively little, while it will be positive if it increases to a greater extent. Graphically, we cannot conclude what will be the final effect of the increase in G on SP. This effect will depend on the level of income that is finally reached.

We proceed to calculate the value analytically. We want to know the value of the increase in the budget balance if there is a higher public expenditure. In that case we know that both G and Y increase with the increase in public spending. Thus, if we start from a level of G = G_0 and a level of Y = Y_0, the initial SP (SP_0) may be defined as:

$SP_0 = tY_0 - G_0 - TR_0$, given the established fiscal policy.

After the increase in expenditure from G_0 to G_1, income will increase to Y_1, so that $Y_1 > Y_0$.

SP then changes its value, and is expressed as follows

$SP_1 = tY_1 - G_1 - TR_0$, (we consider that no other element of fiscal policy changes)

The variation of the SP, can then be expressed as

$$\Delta SP = SP_1 - SP_0 = tY_1 - G_1 - TR_0 - (tY_0 - G_0 - TR_0)$$

In that way

$\Delta SP = tY_1 - tY_0 - G_1 + G_0$

By taking out the common factor of t and grouping the members related to G,

$$\Delta SP = t(Y_1 - Y_0) - (G_1 - G_0) = t\Delta Y - \Delta G$$

The ΔY is the increase in income caused by an increase in public spending. Previously we saw that the value of this increase was equal to

$$\Delta Y = \alpha_G \Delta G = \frac{1}{1-c(1-t)} \Delta G$$

Thus, by substituting the value of this ΔY in the above expression, we can calculate the ΔSP as follows:

$$\Delta SP = t(Y_1 - Y_0) - (G_1 - G_0) = t\Delta Y - \Delta G$$

So, $\Delta SP = t\,\Delta Y - \Delta G = t\frac{1}{1-c(1-t)}\Delta G - \Delta G = \left[\frac{t}{1-c(1-t)} - 1\right]\Delta G$

In this way, by operating

$$\Delta SP = \frac{t-[1-c(1-t)]}{1-c(1-t)}\Delta G = \frac{t-(1-c+ct)}{1-c(1-t)}\Delta G = \frac{t-1+c-ct}{1-c(1-t)}\Delta G$$

Therefore, $\Delta SP = \frac{-(1-c)(1-t)}{1-c(1-t)}\Delta G$

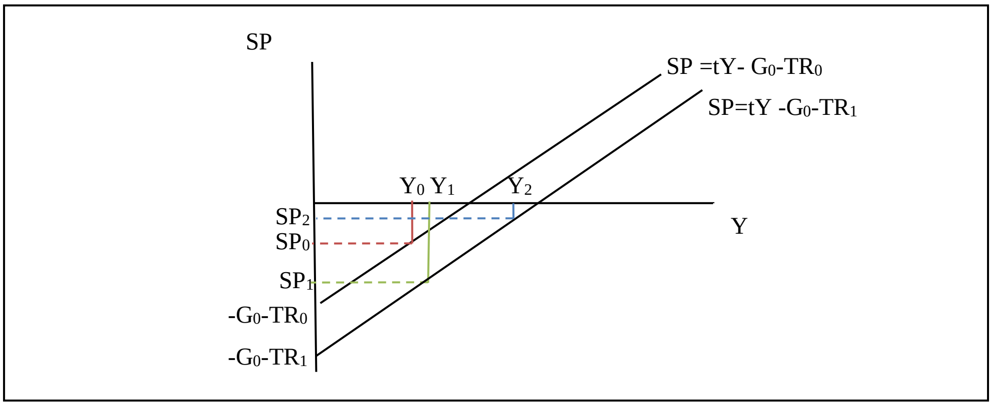
What can be deduced from this result? Since the values of c and t are positive but less than unity, both (1-c) and (1-t), as well as 1-c(1-t) are positive values. Thus, the value of $\frac{-(1-c)(1-t)}{1-c(1-t)}$ is negative. Therefore, a change in public spending causes an effect of opposite sign on the budget balance. If G increases, SP decreases, while if G decreases, SP increases. Thus, an increase in public spending has a negative effect on the budget balance.

It is demonstrated that an increase in public spending reduces the budget surplus. Thus, despite the multiplicative increase in income caused by the increase in G, raising tax revenue, this revenue increases by less than G. The final effect will depend on the value of the increase in G, and on the values of the marginal propensity to consume and the tax rate.

3.6.2. Effects of Public Transfers

An increase in public transfers, i.e., in TR, again has a direct effect on the budget balance, causing it to decrease in value. However, the increase in TR also has a positive effect on income (we have seen above that if TR increases, there is an increase in income equal to $\Delta Y_1 = \alpha_G \, c\Delta G$). If income increases, then we will have higher tax revenues and consequently the budget balance will improve. We then have an opposite effect. On the one hand, if TR increases, SP worsens. But if TR increases, Y grows, tY increases and the SP improves. Which of the two effects is greater? We try to answer this question graphically, through Figure 9.

Figure 9. Effect of an increase in TR on SP.



On this occasion, to illustrate another example, let's assume that we start from an initial situation of negative SP or deficit, given the initial fiscal policy with a given level of TR = TR_0, t and the level of G =G_0. Likewise, we start from an initial situation of income Y equal to Y_0. We can observe that, for this income level, the SP = SP_0, i.e., we are in a situation of public deficit.

What happens if TR increases? If TR increases to TR_1, the value at the origin of public deficit will now be higher, up to $-G_0-TR_1$. We must then draw its value below the previous one. Since the tax rate has not changed, the slope of the SP curve does not change, and the curve is then shifted downward in a parallel pattern.

What is the value of the SP now? The level of income is no longer valid, since the increase in TR causes income to increase. Let us imagine that income increases to Y_1, then, given the new fiscal policy (i.e., the new value of TR), SP is now SP_1 lower than before. We could conclude that if TR increases the SP decreases. However, let us imagine that income increases to Y_2. Then, with the new fiscal policy now SP is SP_2, higher than SP_0. We could conclude that if TR increases the SP increases. Then, what is the final effect? It will be negative if income increases relatively little, while it will be positive if it increases to a greater extent. Graphically, we cannot conclude what will be the final effect of the increase in TR on SP, since this effect will depend on the level of income that is finally reached.

Can we then calculate the value of the increase in the budget balance if there is an increase in TRs? If we start from a level of TR = TR_0 and a level of Y = Y_0, the initial SP, i.e., SP_0, can be defined as

$SP_0 = tY_0 - G_0 - TR_{0,}$ given the fiscal policy established.

After the increase in transfers from TR_0 to TR_1, rent will increase to Y_1, so that $Y_1 > Y_0$.

The SP then changes its value, and is expressed as follows:

$SP_1 = tY_1 - G_0 - TR_{1,}$ (we consider that no other element of fiscal policy changes)

The variation of the SP, can then be expressed as

$\Delta SP = SP_1 - SP_0 = tY_1 - G_0 - TR_1 - (tY_0 - G_0 - TR_0)$

In this way

$\Delta SP = tY_1 - tY_0 - TR_1 + TR_0$

By taking the common factor of t and grouping the related members with TR,

ΔSP = t(Y_1 –Y_0) –(TR_1 –TR_0) = tΔY-ΔTR

The ΔY is the increase in income caused by an increase in public transfers, whose value is

$$\Delta Y = \alpha_G c\, \Delta TR = \frac{c}{1-c(1-t)} \Delta \mathrm{TR}$$

Thus, by substituting the value of this ΔY in the above expression, we can calculate the ΔSP as follows:

$$\Delta SP = t\,\Delta Y - \Delta \mathrm{TR} = \frac{ct}{1-c(1-t)} \Delta \mathrm{TR} \text{ - } \Delta \mathrm{TR} = \left[\frac{ct}{1-c(1-t)} - 1\right] \Delta \mathrm{TR}$$

By operating

$$\Delta \mathrm{SP} = \frac{ct - [1-c(1-t)]}{1-c(1-t)} \Delta TR = \frac{ct - (1-c+ct)}{1-c(1-t)} \Delta TR = \frac{ct - 1 + c - ct}{1-c(1-t)} \Delta TR$$

Therefore,

$$\Delta SP = \frac{-(1-c)}{1-c(1-t)} \Delta \mathrm{TR}$$

What can be deduced from this result? Given that the values of c and t are positive but less than unity, the value of $\frac{-(1-c)}{1-c(1-t)}$ is negative, and therefore changes in transfers have an opposite effect on the budget balance. Thus, an increase in transfers has a negative effect on the budget balance. However, it is also true that a decrease in transfers will lead to an increase in the budget balance.

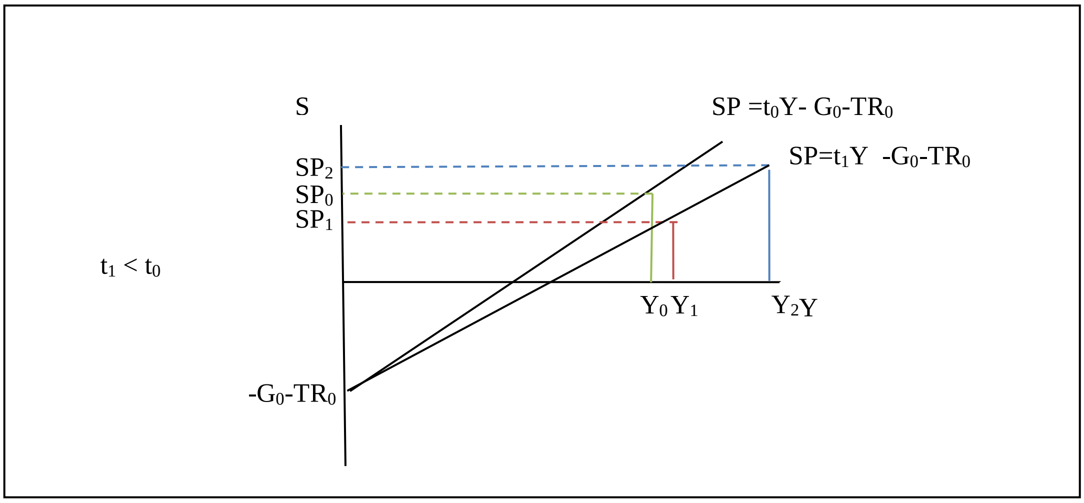
Thus, it has been shown that an increase in public transfers reduces the budget balance. The fact is that, despite the multiplicative increase in income caused by the increase in TR, raising tax collection, this collection increases by less than TR. The final effect will depend on the value of the increase in TR, and on the values of the marginal propensity to consume and the tax rate.

3.6.3. Effects of the Tax Rate

A decrease in the tax rate, i.e., *t*, again has a direct effect on the budget balance, causing it to decrease in value. However, a decrease in *t* also has a positive effect on income (we have seen above that if *t* decreases there is an increase in income equal to $\Delta Y = \frac{-cY_0\Delta t}{1-c(1-t_1)}$). If income increases, then we will have higher tax revenues and consequently the budget balance will improve. We then have an opposite effect. On the one hand, if t decreases, SP worsens (tY falls), but at the same time, when t decreases, Y increases, and therefore tY increases and SP improves. Which of the two effects is greater?

We try to answer this question graphically. Figure 10 shows the effect of an increase in TR on SP.

Figure 10. Effect of a decrease in the tax rate on SP.



Let us assume that we start from an initial situation of positive SP, given the initial fiscal policy with a given level of TR = TR_0, t_0 and the level of G = G_0. Likewise, we start from an initial situation of income Y equal to Y_0. We can observe that for that level of income SP = SP_0. Therefore, we start from a situation of budget surplus.

What happens if t decreases? If t decreases, the value at the origin of the public deficit does not change, since neither the value of G nor the value of T changes. However, as t changes, up to a lower tax rate t_1, the slope of the SP curve decreases, and therefore now the SP curve becomes flatter.

What is the value of SP now? The level of income is no longer valid, since the decrease in *t* causes income to increase. Let us imagine that income increases up to Y_1, then, given the new fiscal policy (i.e., at the new value of *t*), SP is now SP_1 lower than before. We could conclude that, if *t* decreases, SP decreases. However, let us imagine that income increases up to Y_2. Then, with the new fiscal policy, SP is now SP_2, higher than SP_0. We could conclude that, if t decreases, SP increases. So, what is the final effect? It will be negative if income increases relatively little, while it will be positive if it increases to a greater extent. Graphically, we cannot conclude what will be the final effect on the SP of the variation of *t*, since this effect will depend on the level of income that is finally reached.

The influence of a decrease in the tax rate on the budget balance can be examined in the same way as in the previous cases. Recall that a decrease in the tax rate increases the level of income by an amount equal to $\frac{-cY_0\Delta t}{1-c(1-t_1)}$, It might therefore seem that the budget balance also increases, with the level of public spending and transfers remaining constant. However, this is not the case, since a decrease in the tax rate decreases the budget balance despite causing an increase in income, as shown below.

We want to know the value of the change in the budget balance if *t* decreases. In that case we know that *t* decreases, but we also know that Y increases due to the effect that the tax rate has on the level of income. Thus, if we start from a level of TR = TR_0 and a level of Y = Y_0, the initial SP (SP_0) can be defined as:

$SP_0 = t_0Y_0 - G_0 - TR_0$, given the fiscal policy established.

After the decrease of *t* to t_1, the rent will increase to Y_1, so that $Y_1 > Y_0$.

The SP then changes its value, and is expressed as follows:

$SP_1 = t_1Y_1 - G_0 - TR_0$, (we consider that no other element of fiscal policy changes)

The variation of the SP, can then be expressed as

$\Delta SP = SP_1 - SP_0 = t_1Y_1 - G_0 - TR_0 - (t_0Y_0 - G_0 - TR_0)$

In that way

$\Delta SP = t_1Y_1 - t_0Y_0$

We can recall that $\Delta Y = Y_1 - Y_0$, and therefore, $Y_1 = \Delta Y + Y_0$

Thus, substituting the value of Y_1 in the above expression

$$\Delta SP = t_1(\Delta Y + Y_0) - t_0Y_0 = t_1 \ \Delta Y + t_1Y_0 - t_0Y_0$$

If we take the common factor of Y_0

$$\Delta SP = t_1\Delta Y + Y_0(t_1 - t_0) = t_1\Delta Y + Y_0\Delta t$$

We now recall that $\Delta Y = \frac{-cY_0\Delta t}{1-c(1-t_1)}$. Therefore, substituting in the previous expression and taking the common factor of Y_0 and Δt

$$\Delta SP = \left[\frac{-t_1c}{1-c(1-t_1)} + 1\right]Y_0\Delta t = \frac{-t_1c + 1 - c + t_1c}{1 - c(1 - t_1)}Y_0\Delta t$$

Therefore, the variation of the SP is equal to

$$\Delta SP = \frac{(1-c)Y_0\Delta t}{1 - c(1 - t_1)}$$

What can be deduced from this result? Since the values of c and t are positive but less than unity, both (1-c) and 1-c(1-t) are positive values. Thus, changes in the tax rate cause an effect of the same sign on the budget balance. Thus, an increase in the tax rate causes an increase in the budget balance, while a decrease in the tax rate causes a decrease in its value.

Thus, it is demonstrated that a decrease in the tax rate reduces the budget balance. It is important to bear in mind that the effect on this occasion, in addition to depending on the value of c and t_1, also depends on the initial value of income. The higher the initial value of income, the greater the effect that the change in the tax rate will have on the budget balance.

3.7. THE FULL EMPLOYMENT BUDGET BALANCE.

We have seen that the value of the SP depends on fiscal policy. However, we have also seen that, given the same fiscal policy, the SP depends on the income level of the economy. Thus, the value of the SP cannot easily be used to know which fiscal policy is being implemented at a given time.

We can, however, calculate the SP for a given level of income. The full-employment budget is the estimate of public expenditures and revenues if the economic situation of the country were full employment. That is, it is the value of the SP when the income level of the economy is at full employment.

If we denote the full employment income as Y*, the full employment budget balance SP* is equal to

$$SP^* = tY^* - (\overline{G} + \overline{TR})$$

If Y* can be considered constant over a considerable period of time, then SP* can be used to measure or assess what type of fiscal policy is being employed in the country at a given time.

However, we must make some considerations in this regard. First, the SP* is only an estimated value, so it is possible that we can make mistakes when calculating it. Secondly, although it is a rough estimate of the type of policy we are using, we cannot assess whether there are changes among the components of fiscal policy. That is, the SP* will not vary, for example, if G is increased and TR is decreased by the same amount. Therefore, it may not reflect the effect of simultaneous variations in the elements of fiscal policy.

3.8. THE BUDGET BALANCE, SAVINGS AND INVESTMENT.

Earlier we have seen that, in a simple economy, in equilibrium, S=I. But what is their relationship in a closed economy?

In a closed economy, the goods market equilibrium occurs when Y=DA. Also, we know that DA is equal to

DA=C+I+G, so that Y=C+I+G

On the other hand, we also know that in the closed economy income is not equal to disposable income, because now

$YD= Y+TR-T$

We also know that households use their disposable income to save or consume. In other words

$YD=C+S$

So, $YD= C+S = Y+TR-T$

If we replace Y by its value ($Y=C+I+G$), we can now express the above equality as follows

$$C+S= (C+I+G) +TR-T$$

That means, $S=I+G+TR-T$

If the difference between (G+TR)-T is positive, then there is a public deficit (DP). Therefore,

$$S= I+DP$$

In other words, in equilibrium, an economy's savings are no longer equal to investment, but now, in a closed economy, savings are equal to investment plus the public deficit. What does this mean? Household savings will not go entirely to finance the economy's investment operations, but now part of those savings will have to go to finance the public deficit generated by the public sector. The larger the public deficit, the less savings can be used to finance the economy's investment.

3.9. Annex: Public Debt.

Expansionary fiscal policies of increased spending, increased transfers and lower taxes generate reductions in revenues or increases in public spending, worsening the public accounts, i.e., reducing surpluses, or generating public deficits.

When the State spends more than it receives and generates a public deficit, it must borrow to finance this deficit. This borrowing is known as the issuance of new public debt.

The State can borrow from the Central Bank, in which case the debt is monetized by increasing the monetary base (which will be explained in detail in topic 5). It can also borrow from the general public. In this case, the monetary base does not change. The accumulation of these loans is what is known as Public Debt.

3.9.1. The public sector budget constraint

The public sector budget constraint states the following: if a country incurs a public deficit in a period of time t, its public debt, which we can express as B, will increase in that period t.

Thus, the increase in the debt between two periods ($B_t - B_{t-1}$), is equivalent to the public deficit (DP) for that year:

$DP_t = B_t - B_{t-1}$

What is the public deficit? It is the difference between all expenditures incurred by the State minus its revenues. So far, we have defined it as G +TR - T, in a general way. However, in reality G +TR -T is only what is known as the "primary deficit".

In addition, the State incurs other expenses. These are those incurred to meet the interest payments on the Public Debt of previous years. These additional expenses are equal to the product of the real interest rate (r) times the debt held in the previous period (B_{t-1}). These expenses are known as "debt service".

So

$DP_t = (G_t + TR_t) - T_t + rB_{t-1}$

In that way, given that $DP_t = B_t - B_{t-1}$

$B_t - B_{t-1} = (G_t + TR_t) - T_t + rB_{t-1}$

Therefore, if a country has a certain level of initial debt, even if its primary deficit is zero, its debt will continue to increase due to interest payments.

Reordering:

$B_t = (G_t + TR_t) - T_t + rB_{t-1} + B_{t-1} = (G_t + TR_t) - T_t + (1+r)B_{t-1}$

Conclusions of the above expression:

1. Even if the primary deficit remains constant, the existence of public debt raises the deficit and public debt over time.

2. Even if the primary deficit is zero, the existence of public debt raises the deficit due to interest payments and public debt over time.

Is it possible to "stabilize debt"? In other words, is it possible for the debt not to grow, to remain constant?

Given that

$B_t - B_{t-1} = (G_t + TR_t) - T_t + rB_{t-1}$

If the debt does not grow ($B_t - B_{t-1} = 0$), then, $(G_t + TR_t) - T_t + rB_{t-1} = 0$. If we reorder,

$T_t - (G_t + TR_t) = rB_{t-1}$

Thus, in order for the debt not to grow, each period there must be a primary surplus equivalent to the interest payment on past public debt. Therefore, tax revenues must finance all public expenditures ($G_t + TR_t$) and interest payments on public debt.

What can a country do to reduce its public debt? To reduce debt, it must have a primary surplus sufficient to cover interest payments as well as debt repayments. This can be achieved by increasing taxes or reducing public expenditures on goods and services and transfers. It is important to note that the longer it takes to increase taxes or reduce spending to repay the public debt, the larger the accumulated debt will become, and the larger the tax increase or spending reduction will be in the future.

3-9.2. The debt ratio

The debt ratio is the ratio of public debt to a country's GDP. Thus:

Debt ratio = Debt ratio $= \frac{B_t}{y_t}$

Thus, given that

$B_t = (G_t + TR_t) - T_t + (1+r)B_{t-1}$

Debt ratio $= \frac{B_t}{y_t} = \frac{(1+r)B_{t-1}}{y_t} + \frac{(G_t + TR_t) - T_t}{y_t}$

Where $\frac{(G_t + TR_t) - T_t}{y_t}$ is the primary deficit in relation to GDP. That is, the government deficit expressed as a percentage of GDP. If we denote this value as *x*.

Therefore,

$$\text{Debt ratio} = \frac{B_t}{y_t} = \frac{(1+r)B_{t-1}}{y_t} + x$$

Given the above expression, we can multiply and divide the first summand of the expression on the right by y_{t-1}

In that way:

$$\text{Debt ratio} = \frac{B_t}{y_t} = (1+r)\frac{y_{t-1}}{y_t}\frac{B_{t-1}}{y_{t-1}} + x$$

Defining, the growth rate of production, we have that yt-1 /yt = 1/(1 + g). In addition, using the approximation (1 + r) / (1 + g) = 1 + r - g

$$\text{Debt ratio} = \frac{B_t}{y_t} = (1+r-g)\frac{B_{t-1}}{y_{t-1}} + x$$

This value is equivalent to

$$\text{Debt ratio} = \frac{B_t}{y_t} = (r-g)\frac{B_{t-1}}{y_{t-1}} + \frac{B_{t-1}}{y_{t-1}} + x$$

Therefore, it can be stated that

$$\frac{B_t}{y_t} - \frac{B_{t-1}}{y_{t-1}} = (r-g)\frac{B_{t-1}}{y_{t-1}} + x$$

In other words, the growth of the debt ratio depends on two summands:

• The first is the difference between the real interest rate and the GDP growth rate multiplied by the debt rate existing at the end of the previous period. Depending on whether the real interest rate is higher or lower than the real GDP growth rate, this term is a factor that increases or decreases the debt rate. If r>g, the debt rate tends to grow. Whereas if r<g, debt will decrease between the two periods considered.

• The second is the ratio of the primary deficit to GDP. The primary balance in relation to GDP produces a positive or negative effect on debt growth, respectively, in the case of a deficit or a surplus.

It is also important to note that the real interest rate is approximately equal to the difference between the nominal interest rate and inflation. Thus, if we do r = i -π

$$\frac{B_t}{y_t} - \frac{B_{t-1}}{y_{t-1}} = (i - \pi - g)\frac{B_{t-1}}{y_{t-1}} + x$$

So now it can be seen how inflation can contribute to the reduction of the debt rate. However, deflation (decrease in the rate of price variation) has the opposite effect.

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